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AN

ELEMENTARY ARITHME

DESIGNED FOR

Academies and Schools

ALSO

SERVING AS AN INTRODUCTION TO

HIGHER ARITHMETIC.

BY GEORGE R. PERKINS, A. M.,
PRINCIPAL AND PROFESSOR OF MATHEMATICS OF THE NORMAL
[illegible] SCHOOL OF NEW YORK.

PERKINS' SERIES.

Primary Arithmetic, [illegible]
Elementary Arithmetic.
Higher Arithmetic.

Elements of Algeb
Treatise on Algeb
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practical applications.

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NEW YORK; AUTHOR OF TREATISE ON ALGEBRA; ELEMENTS OF ALGEBRA;
HIGHER ARITHMETIC; GEOMETRY, ETC.

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In offering the present edition of Perkins' Elementary Arithmetic to the public, the Publishers desire to call attention to what they deem the peculiar merits of the work.

I. They regard as a prominent feature of the book, the presence throughout of the distinguished mathematical mind of the Author. It is not everything labelled "an explanation," in an Arithmetic, that brings *reasons* to view; nor every operation marked an "analysis" that reveals *principles* or essential relations. There is still a "lower deep" where the ground-matter lies; and this we think Professor Perkins has ploughed up. The examiner may select, at random, proofs of this radical excellence.

We, therefore, believe that the Arithmetic which we submit, is peculiarly adapted to discipline the minds of those who study it, in the science of Numbers, and to advance them to a higher level of intellectual capability; in short, to *train* them *fitly* for advanced departments in Mathematics.

We are confident that the present work will maintain a longer than usual hold on the interest of both teachers and pupils; for it is not, like a cistern, to be exhausted by a few drawings, but like nature's reservoirs, it has the fountain within itself.

II. The Publishers would present as another excellence of the book, its freedom from minute repetitional details which cumber a page, and obstruct a pupil's progress. It is believed that no *principle* is left unelucidated; and that new light is thrown upon many, heretofore imperfectly illustrated. It is regarded as no small merit of the work, that it does not so *dilute principles* and crumble reasons as to enfeeble their power or obscure their clearness. There is such a thing as debilitating a pupil's mind through *excess* of illustration; as inducing a passive reception rather than an active grasp of truths. It is with the intellectual as with the physical system. The digestive process would be less complete, if he who eats should be deprived of the action and the relish of chewing and swallowing his own food; so a true digestion of knowledge requires that the pupil should masticate his own intellectual aliments. We think Professor Perkins' book is happily adapted to secure this result.

III. The general arrangement of the subjects treated is thought to be philosophical. Those are brought into conjunction which are related in idea. The subject of Fractions, of Decimals, of Interest, of Partial Payments, etc., will, in their perspicuousness and their thoroughness, commend themselves to the examiner.

The subject of Proportion and Ratio is presented with peculiar force; as also, in Equation of Payments, the method of finding the Cash Balance.

IV. The method of Extraction of the Cube Root is greatly preferable to the old method. It is far more concise and more comprehensive; saving nearly half the labor, and being applicable, with little variation, to the extraction of *all* roots. The new method is fully and beautifully explained in this work.

V. The properties of the significant figures, and the use of the zero, are, we think, philosophically and concisely presented.

VI. Lastly, we may say, no subject has been omitted on account of any inherent difficulty in elucidating it.

The Publishers take pleasure in the *appearance* of the Book, which certainly *invites* the interest of the scholar.

PREFACE.

It is more than four years since this work was published. During the whole of this time it has been in constant use under my own superintendence; and, consequently, I have had opportunity to ascertain what were its defects, and wherein a difference of arrangement, or other modifications, would be desirable. I have, also, consulted experienced teachers with direct reference to the present revision of the work, and now submit the result to the public.

I am confident that great improvements will be found in the following particulars. In the statements of properties, relations, and principles—in the phraseology of definitions and of Rules—in the methods of illustration—in the order of arrangement of the subjects treated; indeed, throughout the entire work.

My object has been to be concise, yet lucid; to reach the radical relations of numbers; and to present fundamental principles in analyses and examples, that shall leave nothing obscure, yet that shall not embarrass by multiplied processes, or enfeeble by minute details. I hold to the idea that a sufficiency of illustration to lay open thoroughly the subject treated, is all that is desired; and that whatever is *redundant* impairs the force of what is *essential*. Both teachers and pupils will, as I judge, be benefitted by thus leaving them somewhat to the action of their own minds.

It is not easy for me to specify points to which attention may be directed. But I would suggest, the definition of the values of Figures—of the Zero; the illustration of Subtraction; the general treatment of Vulgar Fractions; the introduction of Decimal Fractions before Federal Money; and of Duodecimals immediately after Denominate Decimals; the whole arrangement of Percentage and Interest; the method of finding the Cash Balance in Equation of Payments. And last, but not least, the method of extracting the Cube Root, by means of auxiliary columns. To this method I ask the attention of teachers generally. I believe I have omitted no step necessary to make it perfectly intelligible; and for conciseness and beauty, as well as for practical use, it is incomparably superior to the usual method.

Throughout the entire work many new examples have been given, which have been formed with much care, having the different parts so related as to bring out, when solved, exactly the principle designed. Many of these questions contain statistical and historical facts which it is desirable for all to know, thus giving an interest to the questions which they could not possess in an abstract and simply numerical form.

GEO. R. PERKINS.

UTICA, *March*, 1849.

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ARITHMETIC.

ARTICLE **1.** ARITHMETIC is the science of numbers.

The operations of arithmetic are performed by the aid of five distinct rules, viz.: *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*. These are usually called the FUNDAMENTAL RULES of arithmetic, because all other rules are founded upon them.

What is Arithmetic? How many distinct rules has it for its operations? Repeat their names. What are these usually called? Why are they so called?

NUMERATION.

2. NUMERATION explains the method of *reading* written numbers.

Notation is the writing down of numbers.

Various methods of notation and numeration were used by the ancients. We shall content ourselves with mentioning two, the common or *Arabic* method, and the *Roman* method.

In the common method ten characters are employed. These characters when written are,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

When printed, they become,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

They have the following names:

1 is called One, or a Unit,
2 is called Two, or two Units,
3 is called Three, or three Units,
4 is called Four, or four Units,
5 is called Five, or five Units,
6 is called Six, or six Units,
7 is called Seven, or seven Units,
8 is called Eight, or eight Units,
9 is called Nine, or nine Units,
0 is called Naught, Cipher, or Zero.

Each of these characters, except the *zero*, is called a *digit**; and the first nine, when taken together, are called the *nine digits*.

Any digit is called a *significant* figure.

What is numeration? How is the common method sometimes called? In this method how many characters are employed? What are the names of these characters? What are called digits? What is a significant figure?

3. The significant figures have unchanging values; that is, they always represent *units* or *ones*; but the units which they represent differ in value.

When a significant figure stands disconnected from other figures, the value of its unit is called its *simple* value. When such figure stands in connection with other figures, the value of its unit will depend upon the place which it occupies, and is therefore called its *local* value.

Thus, in the number 3456, which consists of four significant figures standing in connection with each other, each figure expresses units; but units of different values. The right-hand figure, 6, expresses six units, whose value is their simple value; that is, each unit is a *single one.* The second figure, 5, expresses five units; but each unit is ten times greater than each unit of the first figure; therefore the 5 may be read 5 tens, equal to fifty units of simple value. The units expressed by the third figure, 4, are ten times greater than the units expressed by the second figure, and one hundred times greater than those expressed by the first figure; the third figure is therefore read 4 hundreds. The last figure, 3, expresses units ten times greater than the units in 4, and one thousand times greater than the units in 6, and is read 3 thousands.

---

* *From the Latin, digitus,* a finger; because the ancients used to do their reckoning *on their fingers. Originally* 10 was also called a digit.

Hence this property:

When figures are connected in a line as in the number 3456, the units which they express are said to be of different *orders.* Thus, 6 occupies the first place, and its units are of the *first order*, that is, they have their *simple value.* The 5 occupies the second place, and its units are of the *second order*, or *tens.* The 4 occupies the third place, and its units are of the *third order*, or *hundreds.* The 3 occupies the fourth place, and its units are of the *fourth order*, or *thousands.* Hence the above number is *three thousand four hundred and fifty-six.*

To numerate and read the numbers in the following table, proceed thus: Begin with the upper line 3. The first place only being occupied, you numerate Units. Then read, *three* units, or simply *three.* In the second line two places are occupied—then numerate Units, Tens—read *fifty-four.* In the third line three places are occupied; then *numerate* Units, Tens, Hundreds—read two hundred *and sixty-seven*, and so proceed.

NUMERATION TABLE.

Hund. of Mill.	Tens of Mill.	Millions.	Hund. of Th's.	Tens of Th's.	Thousands.	Hundreds.	Tens.	Units.	
Ninth.	*Eighth.*	*Seventh.*	*Sixth.*	*Fifth.*	*Fourth.*	*Third.*	*Second.*	*First.*	PLACE OR ORDER.
.	.	.	.	.	.	.	.	3	Units—*three.*
.	.	.	.	.	.	.	5	4	Units, Tens—*fifty-four.*
.	.	.	.	.	.	2	6	7	Units, Tens, Hundreds —*two hundred and sixty-seven;* and so proceed.
.	.	.	.	.	7	4	3	8	
.	.	.	.	7	2	9	1	2	
.	.	.	4	3	2	6	5	4	
.	.	6	3	5	7	4	2	1	
.	7	6	8	2	4	6	9	8	
6	8	5	7	3	1	9	7	5	

4. In the preceding table no 0 occurs. This character, unlike the digits, represents the *absence* of number. It is used to fill places where no value is to be expressed, and thus to cause the *significant* figures to occupy those places in which they will express the intended values. Thus, 2, standing alone, means 2 units of the first order, or of simple value; but 20 means 2 units of the second order, and no units of the first order; that is, 20 is the expression for 2 tens, or *twenty.* In the same way, 200 means 2 units of the third order, no units of the second order, and no units of the first order; that is, 200 is the expression for *two hundred.*

Hence, a zero placed at the right-hand of a significant figure, increases the simple value of its units tenfold. Two *zeros placed at the* right-hand of a significant figure increase the simple value of its units ten times tenfold, or a hundred-fold. Three zeros a thousand-fold, and so on; every additional zero increases the preceding value tenfold.

In reading numbers containing zeros, we read only the significant figures. Thus the number 20406, consisting of 6 units, no tens, 4 hundreds, no thousands, 2 ten thousands, must be read *twenty thousand four hundred and six.*

Does the value of figures change? What do they always represent? Do their units differ in value? What is the local value of a unit? When significant figures are connected together, what value has the unit of the right-hand figure? What the unit of the second figure, &c.? Give an illustration. When a figure occupies the first place, of what order are its units, &c.? Repeat the Numeration Table. What do you mean by the place of a figure? What by the order of its units? What does the zero represent? For what purpose is it used? What effect has it on the units of the significant figures with which it is connected? What effect have two zeros? What effect has every additional zero? In reading numbers, what use do we make of the zero? What figures do we read?

EXAMPLES.

Numerate and read the annexed numbers:

4
36
205
1237
27835
102007
6300069
54128900
651234567

Also, write down the following numbers under each other, so that units may stand under units, tens under tens, hundreds under hundreds, &c.

Seventy-three.
Three hundred and thirty-seven.
Eight thousand six hundred and one.
Ninety-seven thousand three hundred and forty-three.
Three hundred thousand, five hundred and eleven.
Six millions, one thousand and twenty-five.
Forty-three millions and seventeen.
Two hundred and thirty-three millions and ten thousand.

5. Thus far we have shown how to numerate and *read numbers which do* not contain more than nine places of figures. When there are more than nine places of figures, it will be convenient to divide them into periods of three figures each, as in the following

TABLE.

&c.	&c.	&c.	&c.	&c.
SEPTILLIONS.	Hundreds of Septillions.	27	6	9TH PERIOD.
	Tens of Septillions.	26	5	
	Septillions.	25	7	
SEXTILLIONS.	Hundreds of Sextillions.	24	8	8TH PERIOD.
	Tens of Sextillions.	23	9	
	Sextillions.	22	3	
QUINTILLIONS.	Hundreds of Quintillions.	21	4	7TH PERIOD.
	Tens of Quintillions.	20	2	
	Quintillions.	19	1	
QUADRILLIONS.	Hundreds of Quadrillions.	18	7	6TH PERIOD.
	Tens of Quadrillions.	17	5	
	Quadrillions.	16	3	
TRILLIONS.	Hundreds of Trillions.	15	2	5TH PERIOD.
	Tens of Trillions.	14	1'	
	Trillions.	13	6	
BILLIONS.	Hundreds of Billions.	12	4	4TH PERIOD.
	Tens of Billions.	11	8	
	Billions.	10	5	
MILLIONS.	Hundreds of Millions.	9	3	3D PERIOD.
	Tens of Millions.	8	2	
	Millions.	7	7	
THOUSANDS.	Hundreds of Thousands.	6	4	2D PERIOD.
	Tens of Thousands.	5	8	
	Thousands.	4	9	
UNITS.	Hundreds.	3	4	1ST PERIOD.
	Tens.	2	3	
	Units.	1	2	

By this table we discover that each period, or group of three figures, takes a new name, by which means the numeration of all numbers is made to depend upon that of three figures.

6. The above method of numerating, by giving to each period of three figures an independent name, is due to the *French*. There is another method, sometimes used, called the *English* method. It consists in giving a new name to *each period of six* figures. The *French* way is the simpler, and is generally adopted. We will exhibit the two methods at one view in the following

TABLE.

FRENCH METHOD.		ENGLISH METHOD.
Hundreds of Octillions.	3	Hunds. of Thous. of Quadrillions.
Tens of Octillions.	3	Tens of Thous. of Quadrillions.
Octillions.	3	Thousands of Quadrillions.
Hundreds of Septillions.	3	Hundreds of Quadrillions.
Tens of Septillions.	3	Tens of Quadrillions.
Septillions.	3	Quadrillions.
Hundreds of Sextillions.	3	Hunds. of Thous. of Trillions.
Tens of Sextillions.	3	Tens of Thousands of Trillions.
Sextillions.	3	Thousands of Trillions.
Hundreds of Quintillions.	3	Hundreds of Trillions.
Tens of Quintillions.	3	Tens of Trillions.
Quintillions.	3	Trillions.
Hundreds of Quadrillions.	3	Hunds. of Thous. of Billions.
Tens of Quadrillions.	3	Tens of Thous. of Billions.
Quadrillions.	3	Thousands of Billions
Hundreds of Trillions.	3	Hundreds of Billions.
Tens of Trillions.	3	Tens of Billions.
Trillions.	3	Billions.
Hundreds of Billions.	3	Hunds. of Thous. of Millions.
Tens of Billions.	3	Tens of Thousands of Millions.
Billions.	3	Thousands of Millions.
Hundreds of Millions.	3	Hundreds of Millions.
Tens of Millions.	3	Tens of Millions.
Millions.	3	Millions.
Hundreds of Thousands.	3	Hundreds of Thousands.
Tens of Thousands.	3	Tens of Thousands.
Thousands.	3	Thousands.
Hundreds.	3	Hundreds.
Tens.	3	Tens.
Units.	3	Units.

By the French method of numerating, how many figures are connected in a period? How many do the English connect in a period? Which method is to be preferred?

7. After the pupil has carefully examined this table, let him be required to numerate and read, by dividing into periods of three figures, the following numbers:

1347835674116
3478567321752005
75456278327005717
633456267489136545
45654213400100205437
467743486921785412123456489

Let him also separate them into periods of six figures, according to the English method, and then numerate and read them.

It will be seen, by reference to the foregoing tables, that the *French* and *English* methods of numeration agree as far as nine places of figures, which is as far as we generally wish to extend numbers in the ordinary business operations of life. Numbers could be chosen which should be widely different, and still would be read precisely the same by the two methods. For instance, the French method of reading 103900000000000 is the same as the English method of reading 103000900000000000000, each reading being *one hundred and three trillions, nine hundred billions.*

The same is the case with infinite other numbers which might be selected. Hence the importance of knowing which system of numeration is employed. Twenty billions in the English system is a thousand times twenty billions in the French system.

ROMAN NOTATION.

8. The Romans, as well as many other nations, expressed numbers by certain letters of the alphabet. The Romans made use of only seven capital letters, viz.: I for *one;* V for *five;* X for *ten;* L for *fifty;* C for *one hundred;* D for *five hundred;* M for *one thousand.* The other numbers they expressed by various repetitions and combinations of these letters, as in the following

TABLE.

1 expressed by I.
2 " " II.

As often as any character is repeated, so many

3	expressed	by	III.
4	“	“	IV, or IIII.
5	“	“	V.
6	“	“	VI.
7	“	“	VII.
8	“	“	VIII.
9	“	“	IX.
10	“	“	X.
50	“	“	L.
100	“	“	C.
500	“	“	D.
1000	“	“	M.
2000	“	“	MM.
5000	“	“	$\overline{\text{V}}$.

times is its value repeated.

A less character before a greater, diminishes its value. A less character after a greater, increases its value.

A bar (—) over any number, increases it 1000 fold.

By what means did the Romans express numbers? In this notation, how did repeating a letter affect the value which it represented? How was the value of a character affected when one of less value was placed before it? How when a character of less value was placed after it? How was the value affected by a bar drawn over it?

ADDITION OF SIMPLE NUMBERS.

9. SIMPLE ADDITION is putting together several numbers of the same kind or denomination.

The sum total which is obtained by adding several numbers together, is called the *amount.*

Before explaining the method of adding numbers, we will show the use of the two symbols $=$, $+$.

The symbol $=$, is called the sign of equality, and when *placed between two* quantities, it indicates that they are equal. Thus $1=100 cents, implies that one dollar is equal to one hundred cents.

The symbol +, is called the sign of addition, and when placed between two quantities, indicates that those quantities are to be added. Thus 3+4=7, denotes that the sum of 3 and 4 is equal to 7.

The symbol + is generally read *plus;* a Latin word, meaning more.

What is simple addition? What is the result obtained by adding several numbers together, called? Describe the symbol of equality. Describe that of addition.

By the assistance of these two symbols we may form the following

ADDITION TABLE.

2+0= 2	3+0= 3	4+0= 4	5+0= 5
2+1= 3	3+1= 4	4+1= 5	5+1= 6
2+2= 4	3+2= 5	4+2= 6	5+2= 7
2+3= 5	3+3= 6	4+3= 7	5+3= 8
2+4= 6	3+4= 7	4+4= 8	5+4= 9
2+5= 7	3+5= 8	4+5= 9	5+5=10
2+6= 8	3+6= 9	4+6=10	5+6=11
2+7= 9	3+7=10	4+7=11	5+7=12
2+8=10	3+8=11	4+8=12	5+8=13
2+9=11	3+9=12	4+9=13	5+9=14
6+0= 6	7+0= 7	8+0= 8	9+0= 9
6+1= 7	7+1= 8	8+1= 9	9+1=10
6+2= 8	7+2= 9	8+2=10	9+2=11
6+3= 9	7+3=10	8+3=11	9+3=12
6+4=10	7+4=11	8+4=12	9+4=13
6+5=11	7+5=12	8+5=13	9+5=14
6+6=12	7+6=13	8+6=14	9+6=15
6+7=13	7+7=14	8+7=15	9+7=16
6+8=14	7+8=15	8+8=16	9+8=17
6+9=15	7+9=16	8+9=17	9+9=18

Let the pupil be required to answer the following questions:

4+3=how many?
2+5+1=how many?
5+6+7+2=how many?
8+9+2+1+7=how many?
6+7+5+4+3+2=how many?
1+2+4+3+5+7+6=how many?

EXAMPLES.

1. Where the sums of the several columns are less than ten;—

Add together 2432, 3343 and 4122.

Set the numbers under each other: units under units; tens under tens; hundreds under hundreds; thousands under thousands. Draw a line below the whole.

Add first, the column of units. Set the sum 7 under the column of units; next add the tens; set the sum 9 under the column of tens—next add the hundreds; set the sum 8 under the column of hundreds. Lastly, add the thousands, and set the sum 9 under the column of thousands. The whole amount is, then, nine thousand eight hundred and ninety-seven.

OPERATION.

Thousands.	Hundreds.	Tens.	Units.
2	4	3	2
3	3	4	3
4	1	2	2
9	8	9	7

Add 6264, 2532 and 1203. *Ans.* 9999.
Add 4132, 1001 and 1423. *Ans.* 6556.

2. Where the sums of the several columns equal or *exceed ten;—*

What is the sum total of the following numbers: 3758 4903, 7006, 3713, 3721.

Place the numbers as directed in the preceding example. The sum of the numbers in the units' column is 21—that is, 2 tens and 1 unit. Set the 1 under the units' column, and carry the 2 to the next or tens' column. The sum of the tens' column thus increased is 10 tens; that is, 1 hundred and no tens. Place a zero under the tens' column, and carry the 1 to the hundreds' column. The sum of the hundreds' column, so increased, is 31 hundreds; that is, 3 thousands and 1 hundred. Set the 1 under the hundreds' column, and carry the 3 to the thousands' column. The sum of this column, so increased, is 23 thousands, or 2 tens of thousands and 3 thousands. Set the 3 under the thousands' column, and carry the 2 to the tens of thousands' place; or, what is the same thing, set down the whole of the sum of the last column.

OPERATION.

Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.
	3	7	5	8
	4	9	0	3
	7	0	0	6
	3	7	1	3
	3	7	2	1
2	3	1	0	1

10. From what has now been explained, we know that ten units are equal to one ten, ten tens are equal to one hundred, ten hundreds are equal to one thousand, and so on; ten of any order are equal to one of the next superior order. Hence, for adding numbers of the same denomination, we deduce this

RULE.

I. Place the numbers to be added under each other, so *that units may stand under units, tens under tens, hundreds under hundreds, and so on for the higher orders.*

II. Commencing at the right, find the sum of the numbers in the column of units; if this sum is less than ten, place it immediately under the unit column; but if it equals or exceeds ten, see how many tens it contains, and how many units over; write down the units under the units' column, and carry the tens to the next, or tens' column. In this way proceed with each column, observing to carry for every ten contained in such column, one to the column of the next higher denomination. When we reach the last column, its whole amount must be set down.

How do you write the numbers for addition? Where do you commence to add? If the sum is expressed by a single digit, how do you dispose of it? When it equals or exceeds ten, how do you proceed? What is the rule with regard to carrying? How do you proceed when you come to the last column?

EXAMPLES.

(1.)	(2.)
56430	7921341
12798	82345768
34457	79013265
21325	7890275
125010 amount.	177170649 amount.

PROOF OF ADDITION.

11. The method of proving, or testing the work of addition, is generally to commence at the top of the respective columns and add downwards, carrying one for every ten as before; if the sum is the same as when the columns were added upwards, the work is then supposed *to be correct. This proof* is not infallible, since mistakes may occur in both operations, which shall balance each other.

How is the work of addition generally proved? Is this method of proof infallible? Why not?

(5.)	(6.)	(7.)
34567890	43345678	123423434
2357911	21123355	23785432
234567	27893	9876543
24897	54689	751002
64	734321	10200
37185329	65285936	157846611

8. Add 123405, 2354210, 794327, and 36547, together.
Ans. 3308489.

9. Add 275602, 345607, 4567801, and 365, together.
Ans. 5189375.

10. Add 100375, 406780, 4673005, 4112, and 2478, together.
Ans. 5186750.

11. Add 1034001, 78954, 379205, 367001, and 45637, together.
Ans. 1904798.

12. What is the sum of the following numbers: Three thousand six hundred and fifty, seven thousand eight hundred and thirty-two, eleven thousand five hundred and sixty-seven, ten thousand and fifty-six, four hundred and seventy-two? *Ans.* 33577.

13. What is the sum of the numbers, four thousand three hundred and seventy-three, three thousand one hundred and fourteen, one thousand two hundred and twenty-three, six hundred and fifty-four? *Ans.* 9364.

14. Find the number of days in a year, the days of the respective months being as follows: January, 31, February, 28, March, 31, April, 30, May 31, June, 30, July, 31, August, 31, September, 30, October, 31, November, 30, December, 31.

Ans. 365 days.

15. A man drew five loads of bricks; in the first load he had 1209, in the second load 1453, in the third load 1101, in the fourth load 1212, and in the fifth load 1306. How many bricks were there in all?

Ans. 6278 bricks.

16. If there are shipped from the United States, 15624 barrels of flour to Sweden, 250 barrels to Holland, 205154 barrels to England, 6401 to Texas, 19602 to Mexico, what is the whole amount? *Ans.* 247031 barrels.

17. In 1837 the United States exported 100232 hogsheads of tobacco; in 1838 they exported 100592; in 1839 they exported 78995; in 1840 they exported 119484; in 1841 they exported 147828. How many hogsheads of tobacco were exported during these five years?

Ans. 547131 hogsheads.

18. If the cotton crop of the United States is estimated at 1360532 bales for the year 1839, 2177835 bales for the year 1840, 1634945 bales for the year 1841, and 1683574 bales for the year 1842, how many bales will the four years' crops amount to? *Ans.* 6856886 bales.

19. In 1839 the Onondaga Springs produced 2864718 bushels of salt; in 1840 they produced 2622305 bushels in 1841 they produced 3340769 bushels; in 1842 they produced 2291903 bushels. What is the whole number of bushels during the above four years?

Ans. 11119695 bushels.

20. The United States exported in bullion and specie, in 1838, 3508046 dollars; in 1839, 8776743 dollars; in 1840, 8417014 dollars; in 1841, 10034332 dollars. How much was exported during these four years?

Ans. 30736135 dollars.

21. Amount of tea consumed in the United States, during 1842, was 13482645 pounds; during 1843, it was 12785748 pounds; in 1844, it was 13054327 pounds; in 1845, it was 17162550 pounds; and in 1846 it was 16891020 pounds. What was the whole number of pounds during these five years?

Ans. 73376290 pounds.

22. The amount of coffee consumed in the United States, during the year 1842, was 107383567 pounds; in 1843, it was 85916666 pounds; in 1844, it was 149711820 pounds; in 1845, it was 94358939 pounds, and in 1846 it was 124336054 pounds. What was the whole number of pounds during these five years?

Ans. 561707046 pounds.

23. The number of acres of public land sold by the United States government, in the year 1841, was 1164796 acres; in the year 1842, it was 1129217 acres; in 1843, it was 1605264 acres; in 1844, it was 1754763 acres; and in 1845 it was 1843527 acres. What was the whole number of acres sold during these five years?

Ans. 7497567 acres.

24. The United States revenue for letter postage, under the new law, was as follows: for the year 1842, it was 3953315 dollars; for 1843, it was 3738307 dollars; for 1844, it was 3676162 dollars; and for 1845 it was 3660231 dollars. What was the whole number of dollars during these four years? *Ans*. 15028015 dollars.

25. In 1843, the amount of gold coined at the United States mint and branches, was as follows: At Philadelphia, 4062010 dollars; at the branch at New Orleans, 3177000 dollars; at the branch at Dahlonega, 582782 dollars; at the branch at Charlotte, 287005 dollars. How many dollars of gold coined in all?

Ans. 8108797 dollars.

The sum of the numbers in each row of the following table, whether taken vertically or horizontally, or from corner to corner, is 24156. Let the pupil be required to make these 24 distinct additions.*

TABLE.

2016	4212	1656	3852	1296	3492	936	3132	576	2772	216
252	2052	4248	1692	3888	1332	3528	972	3168	612	2412
2448	288	2088	4284	1728	3924	1368	3564	1008	2808	648
684	2484	324	2124	4320	1764	3960	1404	3204	1044	2844
2880	720	2520	360	2160	4356	1800	3600	1440	3240	1080
1116	2916	756	2556	396	2196	3996	1836	3636	1476	3276
3312	1152	2952	792	2592	36	2232	4032	1872	3672	1512
1548	3348	1188	2988	432	2628	72	2268	4068	1908	3708
3744	1584	3384	828	3024	468	2664	108	2304	4104	1944
1980	3780	1224	3420	864	3060	504	2700	144	2340	4140
4176	1620	3816	1260	3456	900	3096	540	2736	180	2376

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* This table is formed by multiplying the numbers in the magic square of 11, by 36.

The quantity and value of teas and coffee consumed annually, from 1821 to 1846, in the United States, were as follow:

Years.	Teas Consumed.		Coffee Consumed.	
	Pounds.	Value.	Pounds.	Value.
1821	4586223	$1080264	11886063	$2402311
1822	5305588	1160579	18515271	3899042
1823	6474934	1547695	16437045	2835420
1824	7771619	2224203	20797069	2513950
1825	7173740	2346794	20678062	1995892
1826	8482483	3443587	25734784	2710536
1827	3070885	942439	28354197	1130607
1828	6289581	1771993	39156733	3695241
1829	5602795	1531460	33049695	3052020
1830	6873091	1532211	38362687	3180479
1831	4656681	1057528	75700757	5796139
1832	8627144	2081339	36471241	2516120
1833	12927043	4775081	75057906	7525610
1834	13193553	5422275	44346505	4473937
1835	12331638	3594293	91753002	9381689
1836	14484784	4472342	77647300	7667877
1837	14465722	5003401	76044071	7335506
1838	11978744	2559546	82872633	7138010
1839	7748028	1781824	99872517	9006685
1840	16860784	4059545	86297761	7615824
1841	10772087	3075332	109200247	9855273
1842	13482645	3567745	107383567	8447851
1843	12785748	3405627	85916666	5923927
1844	13054327	3152225	149711820	9054298
1845	17162550	4809611	94358939	5380532
1846	16891020	3983337	124336054	7802894
Totals.				

This table will afford material for as many examples in *addition as the* teacher may desire. Thus, he may require the pupil to find the total number of pounds, as well as dollars, for the whole number of years given, or for any particular years within the limits of the table; and as it is very desirable for the pupil to be quick and accurate in the addition of numbers, it will be well for the teacher to extend to considerable length the exercises which may be drawn from the above statistics.

SUBTRACTION OF SIMPLE NUMBERS.

12. SUBTRACTION is taking a less number from a greater.

The greater number is called the *minuend*, and the smaller number is called the *subtrahend;* the result is called the *remainder* or *difference.*

The symbol for subtraction is —. When this symbol is placed between two numbers, it indicates that the second is to be subtracted from the first. Thus, 8—5, denotes that 5 is to be taken from 8. The remainder being 3, we have 8—5=3.

The symbol — is generally read *minus;* a Latin word meaning *less.*

What is Subtraction? What is the greater number called? What is the smaller number called? What is the result called? What symbol is used to denote Subtraction?

By using this symbol, we may form the following

SUBTRACTION TABLE.

2—2= 0	3—3= 0	4—4= 0	5—5= 0
3—2= 1	4—3= 1	5—4= 1	6—5= 1
4—2= 2	5—3= 2	6—4= 2	7—5= 2
5—2= 3	6—3= 3	7—4= 3	8—5= 3
6—2= 4	7—3= 4	8—4= 4	9—5= 4
7—2= 5	8—3= 5	9—4= 5	10—5= 5
8—2= 6	9—3= 6	10—4= 6	11—5= 6
9—2= 7	10—3= 7	11—4= 7	12—5= 7
10—2= 8	11—3= 8	12—4= 8	13—5= 8
11—2= 9	12—3= 9	13—4= 9	14—5= 9
6—6= 0	7—7= 0	8—8= 0	9—9= 0
7—6= 1	8—7= 1	9—8= 1	10—9= 1
8—6= 2	9—7= 2	10—8= 2	11—9= 2
9—6= 3	10—7= 3	11—8= 3	12—9= 3
10—6= 4	11—7= 4	12—8= 4	13—9= 4
11—6= 5	12—7= 5	13—8= 5	14—9= 5
12—6= 6	13—7= 6	14—8= 6	15—9= 6
13—6= 7	14—7= 7	15—8= 7	16—9= 7
14—6= 8	15—7= 8	16—8= 8	17—9= 8
15—6= 9	16—7= 9	17—8= 9	18—9= 9

Let the pupil be required to answer the following questions:

8—2=how many?
11—2=how many?
8—3=how many?
10—3=how many?
12—3=how many?
7—4=how many?
9—4=how many?
11—4=how many?
13—4=how many?

13—5=how many?
11—6=how many?
13—6=how many?
14—7=how many?
16—7=how many?
10—8=how many?
12—8=how many?
13—9=how many?
17—9=how many?

EXAMPLES.

1. In which no figure of the subtrahend is larger th the corresponding figure in the minuend.

From 796 subtract 375.

Place the subtrahend directly under the minuend, so that units may stand under units, tens under tens, hundreds under hundreds.

Then commence at the units' column and subtract—5 from 6 leaves 1; place the one under the units' column, and so proceed with each su ceeding column.

OPERATION.

Hundreds.	Tens.	Units.	
7	9	6	minuend.
3	7	5	subtraher
4	2	1	difference

From 687 subtract 486. *Ans.* 201.

From 7949 subtract 5438. *Ans.* 2511.

From 69975 subtract 59831. *Ans.* 10144.

From 879465 subtract 729355. *Ans.* 150110.

From 987654321 subtract 821350011. *Ans.* 166304310.

2. In which some of the figures of the subtrahend a larger than the corresponding figures of the minuend.

From 867 subtract 496.

OPERATION.

	Hundreds.	Tens.	Units.	
7 hundred, 16 tens, and 7 units,	[7]	[16]	:	
=8 hundred, 6 tens, and 7 units,	$\cancel{8}$	$\cancel{6}$	7	minuend.
	4	9	6	subtraher
	3	7	1	differen

Place the minuend and subtrahend as in the preceding example. Begin at the units' column; 6 from 7 leaves 1. Passing to the tens' figure of the subtrahend, which is 9, we see that it cannot be subtracted from the corresponding figure of the minuend. But we know (ART. **10**,) that 1 of any order is equal to 10 of the next lower order. We therefore take 1 from the hundreds' figure, leaving that figure 7, (which we place in brackets over the 8, marking out the 8,) and counting the 1 hundred as 10 tens, we add it to the 6 tens, making 16 tens, which sum we place in brackets over the 6 and mark out the 6. We now say 9 from 16 leaves 7; 4 from 7 leaves 3.

From 959 subtract 678. *Ans.* 281.
From 767 subtract 349. *Ans.* 418.
From 8965 subtract 7774. *Ans.* 1191.
From 52475 subtract 19304. *Ans.* 33171.

3. We will now give an example of a more difficult operation.

From 8053 subtract 4967.

Place the minuend and subtrahend as before. Commence at the units' column. We cannot subtract the 7 from the 3, as the subtrahend figure is the larger. We therefore take 1 from the tens' figure of the minuend, leaving that figure, 4, (which we place in brackets over the 5, marking out the 5,) and counting the 1 ten as ten units, we

OPERATION.

Thousands.	Hundreds.	Tens.	Units.	
	[9]	[14]		
[7]	[~~10~~]	[~~4~~]	[13]	
~~8~~	~~0~~	~~5~~	~~3~~	minuend.
4	9	6	7	subtrahend.
3	0	8	6	difference.

add it to the 3 units, making 13 units, which sum we place in brackets over the 3 and mark out the 3. We can now subtract the 7 from the 13. We next seek to subtract the 6 from the 4, which we cannot do. We must then seek one from the hundreds' place to be added to the 4. But there are no hundreds there. We then go to the thousands' place. Taking one from the 8, we have 7 left. Place the 7 in brackets over the 8 and mark out the 8. The 1 thousand we carry to the hundreds' place, where it counts 10 hundred; place the 10 over the zero and mark out the 0. Then take 1 hundred from the 10 in the brackets, leaving 9, which, place in second brackets above, and mark out the 10; then add the 1, counting it as 10 tens, to the 4, and you have 14 tens, which place within second brackets over the 4 and mark out the 4.

Now we proceed with the subtraction; 6 from 14 leaves 8; 9 from 9 leaves 0; 4 from 7 leaves 3.

It will be noticed that the minuend appears in three different forms; yet the sum is the same in all. Thus, in the minuend proper, the sum is 8 thousands, 0 hundreds, 5 tens, 3 units; in the minuend in the first brackets, the sum is 7 thousands, 10 hundreds, 4 tens, 13 units; in the second brackets, 7 thousands, 9 hundreds, 14 tens, 13 units: each form being equal to 8053.

NOTE.—The preceding explanations are intended to show the reasons of the process. The pupil should perform similar operations without writing down the steps.

From 8275 subtract 7189.	*Ans.* 1086.
From 6044 subtract 5272.	*Ans.* 772.
From 90000 subtract 1	*Ans.* 89999.

There is another mode, shorter and more practical, for performing subtraction, when figures in the subtrahend are larger than corresponding figures in the minuend.

Take the same example.

OPERATION.

Thousands.	Hundreds.	Tens.	Units.	
8	0	5	3	minuend.
4	9	6	7	subtrahend.
3	0	8	6	difference.

We cannot subtract 7 from 3. Therefore we add 10 to the 3 and say, 7 from 13 leaves 6. Having thus increased the minuend figure 3, by 10 units, we balance that excess by adding 1 ten to the 6 of the subtrahend, making 7 tens. But the 7 tens cannot be subtracted from the 5 tens. Add, then, 10 tens to the 5, making 15 tens, and then say 7 from 15 leaves 8; having added 10 tens to the 5 of the minuend, we restore the balance by adding 1 hundred to the 9 of the subtrahend, making 10 But we cannot subtract 10 from 0. Then we add 10 hundred to the 0, and say 10 from 10 leaves 0. Before subtracting the thousands, we must add 1 to the 4 thousands to compensate for the 10 hundred added to 0, then say 5 from 8 leaves 3.

From 9034 subtract 7941. *Ans.* 1093.
From 8087 subtract 4759. *Ans.* 3328.
From 87315 subtract 19848. *Ans.* 67467.
From 64281 subtract 38796. *Ans.* 25485.

From what has been done, we deduce this

RULE.

I. Place the subtrahend under the minuend, so that units may stand directly under units, tens under tens, &c.

II. Then commencing at the right, subtract each figure of the subtrahend from the corresponding figure of the minuend; observing, when a figure of the subtrahend is greater than the corresponding figure of the minuend, to increase the minuend figure by 10 *before subtracting, and then to carry* 1 *to the next figure of the subtrahend.*

How do you place the numbers for subtraction? Where do you commence to subtract? Explain the method of subtracting when the figure in the subtrahend exceeds the corresponding figure of the minuend.

EXAMPLES.

4. From 34678 subtract 13787.

OPERATION.

34678
13787
20891 difference.

(5.)		(6.)	
789347		10345678937	
120305		902134124	
669042	difference.	9443544813	difference.

PROOF OF SUBTRACTION.

12. If the operation is rightly performed, the difference added to the subtrahend must equal the minuend.

	(7.)	(8.)	(9.)
	78543	612045	9345678201
	23056	137891	3279609167
Differences.	55487	474154	6066069034
Proofs.	78543	612045	9345678201

10. From seven million three hundred and sixty-five thousand, two hundred and thirty-nine, take three hundred and forty-two thousand and thirteen.

Ans. 7023226.

11. From one million and eleven, subtract thirteen.

Ans. 999998.

12. From three hundred and sixty-five thousand, take three hundred and sixty-five

Ans. 364635.

13. America was discovered in 1492. How many years from that time to the year 1844?

Ans. 352 years.

14. If a man receive 11345 dollars, and pay out of it 9203 dollars, how much will he have remaining?

Ans. 2142 dollars.

15. In 1842 the Onondaga Salt Springs yielded 2291903 bushels of salt, and in 1826 they yielded 827505 bushels. How many more bushels were produced in 1842 than in 1826?

Ans. 1464398 bushels.

16. In 1842 the United States shipped to England 205154 barrels of flour, to Scotland 3830 barrels. How many more barrels were sent to England than to Scotland?

Ans. 201324 barrels.

17. Two men start together from the same place, and travel in the same direction; one goes 63 miles each day, and the other goes 37 miles. How far apart will they be at the end of the first day?

Ans. 26 miles.

18. George Washington was born in the year 1732; he died in the year 1799. To what age did he live?

Ans. 67 years.

19. At an election 12572 votes are taken, of which the successful candidate received 7391. How many votes did the other candidate receive?

Ans. 5181 votes.

20. And what was the first one's majority?

Ans. 2210 votes.

21. The coinage of the United States mint for 1843 was in value 11967830 dollars, and in 1846 it was 6633965 dollars. How much greater in value was the coinage in 1843 than in 1846?

Ans. 5333865 dollars.

22. The total number of pieces coined in 1843 was 114640582, and in 1844 it was 9051834. How many more pieces were coined in 1843 than in 1844?

Ans. 105588748 pieces.

23. In the year 1846, the value of the gold coin produced at the mint was 4034177 dollars; the value of the silver coin was 2558580 dollars; and the copper coin was 41208 dollars. How much greater was the value of the gold than the silver, and how much greater the copper? Also, how much did the silver exceed the copper?

Ans. {
Gold exceeded silver by 1475597 dollars.
" " copper " 3992969 "
Silver " " " 2517372 "

24. In 1835, the number of post offices in the United States was 10770; extent of post roads 112774 miles; in 1845, the number of offices was 14183; and extent of roads 143940 miles. How many offices were added during these 10 years, and how many additional miles of road were added?

Ans. {
3413 post offices.
31166 miles of road.

25. In 1840 the population of New York was 2428921, and in 1830 it was 1913006. What was the increase during this 10 years? *Ans.* 515915.

QUESTIONS INVOLVING ADDITION AND SUBTRACTION.

1. A lets B have 60 bushels of wheat, worth 70 dollars, a fine horse worth 150 dollars, and 37 dollars' worth of butter. B in turn gives A his note for 110 dollars, and the rest in cash. What is the amount of cash?
Ans. 147 dollars.

2. A borrows of B, at one time, 375 dollars; at a second time he borrows 95 dollars, and at a third time he borrows 413 dollars; he has paid him 319 dollars. How much does he still owe him? *Ans.* 564 dollars.

3. A person left a fortune of 10573 dollars to be divided between two sons and one daughter; the first son received 4309 dollars, the other son had 4987 dollars. How much did the daughter receive?
Ans. 1277 dollars.

4. Two persons are 375 miles apart; they travel towards each other; at the end of one day, one has travelled 93 miles, and the other 57 miles. How far apart are they?
Ans. 225 miles.

5. A farmer sold a span of horses for 150 dollars, a cow for 27 dollars, some cheese for 83 dollars, and 7 tons of hay for 56 dollars. He purchased 10 yards of broadcloth worth 45 dollars, a cook stove for 23 dollars, and a pleasure carriage for 80 dollars. How much money will he have left? *Ans.* 168 dollars.

6. In the year 1840, the coinage of the United States mint was as follows: 1675302 dollars of gold, 1726703

dollars of silver, and 24627 dollars of copper. In the year 1841 the gold coin amounted to 1091597, the silver to 1132750, and the copper to 15973. How much was the whole value for each year? How much greater was the whole coinage in 1840 than in 1841? In each year, how much greater was the value of the silver than that of the gold and copper respectively?

Ans.
In 1840 total value was $3426632.
" 1841 " " " 2240320.
" 1840 exceeded 1841 by 1186312.
" 1840 silver exceeded gold by 51401.
" " " copper " 1702076.
" 1841 silver exceeded gold by 41153.
" " " copper " 1116777.

MULTIPLICATION OF SIMPLE NUMBERS.

14. Multiplication teaches to repeat one of two numbers as many times as there are units in the other.

The number to be repeated is called the *multiplicand.*

The number denoting how many times the multiplicand is to be repeated, is called the *multiplier.*

Both multiplicand and multiplier are called *factors.**

The result obtained is called the *product.*

The symbol for multiplication is × ; this written between two numbers, indicates that they are to be multiplied together. Thus, 3 × 7 denotes that 3 is to be repeated 7 times, or, which is the same thing, 7 is to be repeated 3 times.

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* *From a Latin word, meaning to make;* because, multiplied together, they make *the product.*

By the assistance of this symbol, we may form the following

MULTIPLICATION TABLE.[*]

2×0= 0	4×0= 0	6×0= 0	8×0= 0
2×1= 2	4×1= 4	6×1= 6	8×1= 8
2×2= 4	4×2= 8	6×2=12	8×2=16
2×3= 6	4×3=12	6×3=18	8×3=24
2×4= 8	4×4=16	6×4=24	8×4=32
2×5=10	4×5=20	6×5=30	8×5=40
2×6=12	4×6=24	6×6=36	8×6=48
2×7=14	4×7=28	6×7=42	8×7=56
2×8=16	4×8=32	6×8=48	8×8=64
2×9=18	4×9=36	6×9=54	8×9=72
3×0= 0	5×0= 0	7×0= 0	9×0= 0
3×1= 3	5×1= 5	7×1= 7	9×1= 9
3×2= 6	5×2=10	7×2=14	9×2=18
3×3= 9	5×3=15	7×3=21	9×3=27
3×4=12	5×4=20	7×4=28	9×4=36
3×5=15	5×5=25	7×5=35	9×5=45
3×6=18	5×6=30	7×6=42	9×6=54
3×7=21	5×7=35	7×7=49	9×7=63
3×8=24	5×8=40	7×8=56	9×8=72
3×9=27	5×9=45	7×9=63	9×9=81

The foregoing table should be committed to memory by the pupil.

---

[*] This table uses no factor consisting of more than one digit. I am aware that many tables of this kind are extended as far as 12 times 12, and others as far as 25 times 25, and even further; but I see no good reason why it should terminate at 12 times 12, any more than 13 times 13. I have therefore thought it better to limit it to 9 times 9, this being as far as it can extend by using but one digit as a factor. Still I have no objection to pupils committing to memory the products of as large factors *as they may wish.*

15. *The pupil must also bear in mind that the multiplier and multiplicand may be interchanged without altering the product.* Thus:

$4\times8=8\times4=32 \qquad 4\times6=6\times4=24$

$9\times7=7\times9=63 \qquad 3\times5=5\times3=15.$

What does multiplication teach? The number to be repeated is called what? The number denoting how many times the multiplicand is to be repeated is called what? What are the multiplicand and multiplier sometimes called? The result obtained is called what? What is the symbol for multiplication? Can the multiplier and multiplicand exchange places without altering the product?

When the multiplicand consists of more than one figure, and the multiplier has but one figure, we proceed as follows:

Multiply 697 by 3.

Place the multiplier under the multiplicand, units under units. First, multiply the 7 units by the 3 units; we obtain 21 units, or 2 tens and 1 unit. Write the 1 under the unit column, and the 2 under the tens' column. Next, multiply the 9 tens by the 3, and we have 27 tens; equal to 2 hundred and 7 tens. Write the 7 tens under the tens' column, and the 2 under the hundreds' column. Finally, multiply the 6 hundreds by the 3 and we have 18 hundreds, which is the same as 1 thousand and 8 hundreds. Write down the 8 under the hundreds' column and carry the 1 to the thou*sands' place; that is,* write down the whole 18

OPERATION.

Thousands.	Hundreds.	Tens.	Units.	
	6	9	7	multiplicand.
			3	multiplier.
		2	1	units.
	2	7		tens.
1	8			hundreds.
2	0	9	1	product.

We then add these partial products, and obtain 2091 for the total product.

By recalling to mind (ART. 10,) that ten in the place of units are equal to one in the place of tens, ten in the tens' place are equal to one in the hundreds' place, &c., we may perform the above multiplication as follows:

First, multiplying 7 of the multiplicand by 3 the multiplier, we obtain 21 units, which are the same as 2 tens and 1 unit. Hence we write down the 1 under the units' column, and reserve the 2 to carry to the tens'. Next, multiplying the 9 by 3, we find 27 tens, to which, adding the 2 tens reserved, we have 29 tens, which are equal to 2 hundreds and 9 tens. Write down the 9 under the tens' column, and reserve the 2 to carry to the hundreds. Finally, multiplying the 6 by 3, we have 18 hundreds; to which add the 2 hundreds reserved, and we have 20 hundreds, the whole of which we write down, obtaining 2093 for the product.

OPERATION.

697 multiplicand.
3 multiplier.
2091 product.

Again, let it be required to multiply 367 by 84. Here the multiplier consists of more than one figure.

Place the multiplier under the multiplicand, units under units, and tens under tens.

Multiplying first by the 4 units, we find 1468 for the product. We are next to multiply by the 8 tens. Now, it is obvious that 1 unit, taken ten times, that is, multiplied by 1

OPERATION.

367 multiplicand.
84 multiplier.
1468
2936
30828 product.

ten, must produce 10 units or 1 ten. So 7 units, (as in the example,) multiplied by 8 tens, must produce 56 tens, or 5 hundreds and 6 tens. Therefore, set the first figure, 6 of this second product under the tens' column and reserve the 5 to carry to the hundreds. The next step is the multiplication of tens by tens, which must produce hundreds, since 1 ten, taken 1 ten times, is equal to 1 hundred. Therefore 8 tens times 6 tens are 48 hundreds; to which add the 5 hundreds reserved, and we obtain 53 hundreds; equal to 5 thousands and 3 hundreds. Place the 3 under the hundreds' column, and carry the 5 to the next column, and so proceed throughout. The sum of these partial products will give the total product, 30828.

If the multiplier consists of three figures, its left-hand or hundreds figure, multiplied into the units of the multiplicand, will give hundreds for the first figure of the product, which must of course be set down under the hundreds' column; hundreds and tens, multiplied together, will give thousands; hundreds and hundreds multiplied together will give ten thousands, &c.

If the multiplier consists of four figures, its left-hand or thousands' figure multiplied into units, will give thousands for the first figure of the product, which must be set down under the thousands' column. Thousands multiplied into tens, gives tens of thousands; into hundreds, gives hundreds of thousands; and so on.

It would be necessary to annex ciphers to the figures in these several products, to show their true places, if these places were not determined by the position of the figures with relation to other figures, whose places are known.

16. If we again take the first example, which is to multiply 697 by 3, we remark that since 697 is to be repeated 3 times, it may be done by writing it down 3 times, and then adding, thus:

$$\begin{array}{r} 697 \\ 697 \\ 697 \\ \hline 2091 \\ \hline \end{array}$$

And it is obvious that all questions of multiplication may be performed by addition.

Hence, multiplication is sometimes defined as being a concise way of performing several additions.

Note.—When a zero or 0 occurs in the multiplier, we may observe that its product must remain 0, since nothing repeated any number of times is still nothing.

PROOF OF MULTIPLICATION.

17. If we interchange the multiplier and multiplicand, and then multiply, we shall obtain the same product if the work is right. (See Art. **15.**)

As in addition, these two results may be alike, and still the work may be wrong, since mistakes may occur in both operations. As good proof as any, is to carefully repeat the multiplication.

When 0 is multiplied by any number, what is the result? How is multiplication sometimes defined. How may multiplication be proved? Is this method infallible? Why not? What is as good proof as any other?

CASE I.

18. When the multiplier consists of only one figure.

From what has already been done, we deduce this

RULE.

Place the multiplier under the unit figure of the multiplicand. Draw a horizontal line underneath.

Then multiply each figure of the multiplicand by the multiplier, observing to carry one for every ten, as in addition.

When the multiplier consists of but one figure, how do you proceed? What rule do you observe in carrying?

EXAMPLES.

(1.)	(2.)	(3.)
1234	234156	612378
2	3	4
2468	702468	2449512

(4.)	(5.)	(6.)
897654	1003456	205670678
5	6	7
4488270	6020736	1439694746

(7.)	(8.)
6531023456	891030756078
8	9
52248187648	8019276804702

CASE II.

19. When the multiplier consists of more than
figure.

RULE.

I. Place the multiplier under the multiplicand, so that units may stand under units, tens under tens, hundreds under hundreds, &c.

II. Multiply successively by each figure of the multiplier, as in Case I., observing to place the right-hand figure of each partial product directly under the figure multiplied by.

III. Then add together these partial products, and the sum will be the total product sought.

When the multiplier consists of more than one figure, how do you write it? How do you then multiply? How do you add up?

EXAMPLES.

(1.)	(2.)	(3.)
23474	4567031	4005604
23	147	123
70422	31969217	12016812
46948	18268124	8011208
539902	4567031	4005604
	671353557	492689292

4. Multiply 12345 by 12. *Ans.* 148140.
5. Multiply 23456 by 11. *Ans.* 258016.
6. Multiply 34567 by 13. *Ans.* 449371.
7. Multiply 780056 by 21. *Ans.* 16381176.
8. Multiply 6503456 by 234. *Ans.* 1521808704.
9. Multiply 3471032 by 70056. *Ans.* 243166617792.
10. Multiply 1240578 by 302014.
Ans. 374671924092.

11. Multiply 235678 by 753465.

Ans. 177575124270.

12. Multiply 98610275 by 35789.

Ans. 3529163131975.

CASE III.

20. When the multiplier, or multiplicand, or both, have one or more ciphers at the right.

We know from what has been said, (Art. **4**,) that multiplying by 10 is the same as annexing a cipher to the right of the figure or sum to be multiplied; multiplying by 100 is the same as annexing two ciphers to the right of the figure or sum to be multiplied, &c.

Hence we deduce this

RULE.

Multiply by the significant figures,(as in Case II.) and to the product annex as many ciphers as there are in both multiplier and multiplicand.

When there are ciphers at the right of the multiplier, or multiplicand, or both, how do you proceed?

EXAMPLES

1. Multiply 365 by 10. *Ans.* 3650
2. Multiply 12040 by 100. *Ans.* 1204000.
3. Multiply 204500 by 3000. *Ans.* 613500000.
4. Multiply 7003000 by 240000.

Ans. 1680720000000.

5. Multiply 307210000 by 3780000.

Ans. 1161253800000000.

CASE IV.

21. When the multiplier is a composite number.

A composite number is one which may be produced by multiplying two or more numbers together. Thus: 35 is a composite number, which may be produced by multiplying 5 and 7 together.

The 5 and 7 are called the *factors* or component parts of 35.

The factors of 12, are 3 and 4, or 2 and 6.

Suppose we wish to multiply 48 by 35.

If we first multiply 48 by 5, we find 240 for the product; if now we multiply this product by 7, we obtain 1680, which is evidently the same as 35 times 48.

Hence we infer this

RULE.

Multiply the sum given by one of the factors, and this product by another factor, and so on, until all the factors are used. The last product will be the one sought.

EXAMPLES.

1. Multiply 365 by 28.

The factors of 28 are 4 and 7. Hence we have this

OPERATION.

$$\begin{array}{rl} 365 & \\ 4 & \text{one of the component parts.} \\ \hline 1460 & \\ 7 & \text{the other component part.} \\ \hline 10220 & \textit{Ans.} \end{array}$$

2. Multiply 374 by 24 = 4 × 6 = 3 × 8 = 2 × 12 = 2 × 3 × 4.

FIRST OPERATION.

374	
4	1st component part.
1496	
6	2d component part.
Ans. 8976	

SECOND OPERATION.

374	
3	1st component part
1122	
8	2d component part.
Ans. 8976	

THIRD OPERATION.

374	
2	1st component part.
748	
12	2d component part.
1496	
748	
Ans. 8976	

FOURTH OPERATION.

374	
2	1st component part.
748	
3	2d component part.
2244	
4	3d component part.
Ans. 8976	

From the above examples, we see that it makes no difference how we resolve the multiplier into factors, provided we multiply in succession by all the factors.

What is a composite number? What are the component parts? How do you proceed when the multiplier is a composite number? Does it make any difference which component part we first multiply by?

3. Multiply 345678 by 36 = 6 × 6 = 4 × 9 = 3 × 12 = 3 × 3 × 4. Ans. 12444408.

4. Multiply 1002456 by $72 = 8 \times 9 = 2 \times 3 \times 3 \times 4 = 2 \times 2 \times 2 \times 3 \times 3$. *Ans.* 72176832.

5. Multiply 7540102 by $84 = 7 \times 12 = 3 \times 4 \times 7 = 2 \times 2 \times 3 \times 7$. *Ans.* 633368568.

EXERCISES IN MULTIPLICATION.

1. Suppose I buy 15 loads of bricks, each load containing 1250 bricks, how many bricks have I?

Ans. 18750 bricks.

2. In an orchard there are 107 apple-trees, each producing 19 bushels of apples. How many bushels does the whole orchard yield? *Ans.* 2033 bushels.

3. If a person travel 17 days at the rate of 37 miles each day, how many miles will he travel in all?

Ans. 629 miles.

4. If a person buy 175 barrels of salt, each weighing 304 pounds, how many pounds in all will he have?

Ans. 53200 pounds.

5. Suppose I purchase the following bill of merchandise:

3 Firkins of butter, each 15 dollars.
7 Hogsheads of molasses, each 23 dollars.
12 Bags of coffee, each 11 dollars.
5 Boxes of raisins, each 2 dollars.
3 Boxes of lemons, each 5 dollars.

How many dollars must I give for the whole?

Ans. 363 dollars.

6. How many dollars will the following bill of goods amount to?

52 Yards of black broadcloth, at 4 dollars per yard.
40 Yards of Brussels carpeting, at 2 dollars per yard.
2 Sofas, each 56 dollars.

9 Mahogany chairs, each 5 dollars.
5 French bedsteads, each 7 dollars.

Ans. 480 dollars.

7. If the railroad extending between Albany and Buffalo, a distance of 326 miles, cost 25649 dollars per mile, what was the entire cost? *Ans.* 8361574 dollars.

8. How many bushels of potatoes may be produced from 13 acres of land, if each acre produces 212 bushels? *Ans.* 2756 bushels.

9. How much must be paid for constructing 18 miles of plank-road, at 4211 dollars per mile? *Ans.* 75798 dollars.

10. How much will 543 cords of wood cost, at 5 dollars per cord? *Ans.* 2715 dollars.

11. In one year there are 8766 hours, how many hours in 1848 years? *Ans.* 16199568 hours.

12. In one cubic foot there are 1728 cubic inches, how many cubic inches in 17 cords of wood, each cord containing 128 cubic feet? *Ans.* 3760128 cubic inches.

13. What will 13 square miles of land cost, at 17 dollars per acre, there being 640 acres in one mile? *Ans.* 141440 dollars.

14. How many miles will a steam locomotive pass in 7 days of 24 hours each, if it move at the rate of 45 miles each hour? *Ans.* 7560 miles.

15. If the earth move in its orbit 68000 miles per hour, how far will it move in 365 days of 24 hours each? *Ans.* 595680000 miles.

16. If one mile of railroad require 116 tons of iron, worth 53 dollars per ton, what will be the cost of sufficient iron to construct a road of 78 miles in length? *Ans.* 479544 dollars.

17. In an orchard of 105 apple trees the average pro-

duce of each tree is 7 barrels of fruit, worth 3 dollars per barrel. What was the income of the orchard?

Ans. 2205 dollars.

DIVISION OF SIMPLE NUMBERS.

22. DIVISION teaches the method of finding how many times one number is contained in another.

The number to be divided is called the *dividend.*

The number by which we divide is called the *divisor.*

The number of times which the dividend contains the divisor is called the *quotient.*

Besides these three parts there is sometimes a *remainder*, which is of the same name as the dividend, since it is a part of it.

The sign usually employed to indicate division is ÷. Thus, 12 ÷ 3, denotes that 12 is to be divided by 3.

By using this sign we may form the following

DIVISION TABLE.

2÷2=1	3÷3=1	4÷4=1	5÷5=1
4÷2=2	6÷3=2	8÷4=2	10÷5=2
6÷2=3	9÷3=3	12÷4=3	15÷5=3
8÷2=4	12÷3=4	16÷4=4	20÷5=4
10÷2=5	15÷3=5	20÷4=5	25÷5=5
12÷2=6	18÷3=6	24÷4=6	30÷5=6
14÷2=7	21÷3=7	28÷4=7	35÷5=7
16÷2=8	24÷3=8	32÷4=8	40÷5=8
18÷2=9	27÷3=9	36÷4=9	45÷5=9

DIVISION TABLE.—(*Continued.*)

6÷6=1 12÷6=2 18÷6=3 24÷6=4 30÷6=5 36÷6=6 42÷6=7 48÷6=8 54÷6=9	7÷7=1 14÷7=2 21÷7=3 28÷7=4 35÷7=5 42÷7=6 49÷7=7 56÷7=8 63÷7=9	8÷8=1 16÷8=2 24÷8=3 32÷8=4 40÷8=5 48÷8=6 56÷8=7 64÷8=8 72÷8=9	9÷9=1 18÷9=2 27÷9=3 36÷9=4 45÷9=5 54÷9=6 63÷9=7 72÷9=8 81÷9=9

23. Division may also be represented by placing the divisor under the dividend, with a short horizontal line between them; thus, $\frac{10}{2}$ denotes that 10 is to be divided by 2. In the same way we have

$\frac{12}{2}=12\div2$; $\frac{13}{3}=13\div3$; $\frac{17}{5}=17\div5$; $\frac{53}{7}=53\div7$.

This method is employed, when in division there is a remainder, to express accurately the value of the quotient.

What does division teach? What is the number to be divided called? What is the number by which we divide called? What is the number of times which the dividend contains the divisor called? There is sometimes another part, what is it? Of what name is the remainder? What is the symbol of division? By what other method is division denoted?

When the divisor consists of only one figure, we proceed as follows:

Divide 973 by 7.

Having placed the divisor at the left of the dividend, keeping them separate by means of a curved line, we draw a straight horizontal line *underneath.*

OPERATION.

7)973

139 quotient.

We then say, 7 is contained in 9, 1 time and 2 remainder; we write the 1 underneath. As the 9 occupies the hundreds' place, the 2 remainder must be 2 hundreds. The next figure, 7, to be divided, is tens, to which we add the 2 hundreds, or 20 tens, making 27 tens; which result is obtained by prefixing the 2 to the 7. Next, we see how many times 7 is contained in 27, which is 3 times and 6 remainder; we place the 3 for the next figure of the quotient, and conceive the 6 to be prefixed to the next figure of the dividend, making 63; which is the same as adding 6 tens or 60 units to the 3 units. Finally, we find 7 is contained in 63, 9 times.

Thus 7 is contained 139 times in 973. Hence, 139 repeated 7 times must equal 973.

24. Suppose we wish to know how many times 8 is contained in 32. We might proceed as follows: since 32 is greater than 8, we know that 8 is contained in it, at least once; therefore, subtracting 8 from 32, we find 24 for a remainder. Again, we know that 8 is contained at least once in 24; therefore, subtracting 8 from 24, we have 16, from which, subtracting 8, we have left 8; finally, from 8 subtracting 8, we have no remainder. Hence, we perceive that 8 has been subtracted 4 times from 32, that is, 8 is contained just four times in 32. It is obvious that by continued subtractions any operation in division may be performed.

For this reason division is said to be a concise way of performing several subtractions.

CASE I.

25. Short Division is the method of operation when *the divisor consists* of only one figure.

From the preceding operation we infer the following

RULE.

I. Place the divisor at the left of the dividend, keeping them separate by a curved line, and draw a straight line underneath the dividend.

II. Seek how many times the divisor is contained in the left-hand figure or figures of the dividend, and place the result directly beneath, for the first figure of the quotient.

III. If there is no remainder, divide the next figure of the dividend for the next figure of the quotient. But when there is a remainder, conceive it to be prefixed to the next succeeding figure of the dividend before making the next division. If a figure of the dividend, which is required to be divided, is less than the divisor, we must write 0 *in the quotient, and consider that figure as a remainder.*

Division is said to be a concise way of performing what? What is Short Division? Repeat the rule.

EXAMPLES.

1. Divide 2345675 by 8.

OPERATION.

Divisor 8)2345675 dividend.

Quotient 293209 with 3 remainder.

26. When there is a remainder, we may place it over the divisor, with a short horizontal line between them, thus indicating that this remainder is still to be divided by the divisor, agreeably to Art. **23.**

2. Divide 12456789 by 4. *Ans.* 3114197$\frac{1}{4}$.
3. Divide 78900346 by 7. *Ans.* 11271478.
4. Divide 131305678 by 6. *Ans.* 21884279$\frac{4}{6}$.
5. Divide 357020348 by 3. *Ans.* 119006782$\frac{2}{3}$.

CASE II.

27. Long Division is the method of operation when the divisor consists of more than one figure.

EXAMPLES.

1. Divide 4703598 by 354.

It requires 3 figures, 470, of the dividend to contain the divisor 354. This is contained once in 470; we place the 1 at the right of the dividend for the first figure of the quotient, keeping it separate from the dividend by a curved line. Multiplying the divisor by this quotient figure, and subtracting the product from 470, we have 116 for a remainder, to which we annex the next figure, 3, of the dividend, thus forming the number 1163. We now seek how many times the divisor is contained in 1163, which is 3 times. We place the 3 for a second figure of the quotient. Multiplying the divisor.

OPERATION.

```
Divisor. Dividend. Quotient.
354) 4703598 (13287
     354  first product.
     ----
     1163
     1062 second product.
     ----
      1015
       708 third product.
       ----
       3079
       2832 fourth product.
       ----
        2478
        2478 fifth product.
        ----
```

by this second figure, and subtracting the product from 1163, we find 101 for a second remainder; to which annexing 5, the next figure of the dividend, we have 1015. Thus we proceed till all the figures of the dividend have been brought down.

From the above work we readily deduce the following

RULE.

I. Place the divisor at the left of the dividend, keeping them separate by a curved line.

II. Seek how many times the divisor is contained in the fewest figures of the dividend that will contain it; set the figure expressing the number of times at the right of the dividend for the first figure of the quotient, keeping dividend and quotient separate by means of a curved line.

III. Multiply the divisor by this quotient figure, and subtract the product from those figures of the dividend used, and to the remainder annex the next figure of the dividend; then find how many times the divisor is contained in this new number, and write the result in the quotient.

IV. Again, multiply the divisor by this last quotient figure, and subtract the product from the last number which was divided, and to the remainder annex the next figure of the dividend. Thus continue the operation until all the figures of the dividend have been brought down.

NOTE 1.—Having brought down a new figure, if the number thus formed be less than the divisor, it will contain it 0 times; we therefore write 0 in the quotient, and bring down another figure.

NOTE 2.—If in multiplying the divisor by any quotient figure we obtain a product which exceeds the number we sought to divide, we must make the quotient figure smaller.

NOTE 3.—If a remainder should be found larger than the divisor, the *quotient figure* must be taken larger.

28. If, now, taking the preceding example, we multiply the divisor by the quotient, we shall have this

OPERATION.

```
    354
  13287
  -----
   2478   first product.
  2832    second product.
  708     third product.
1062      fourth product.
354       fifth product.
-------
4703598
-------
```

Here we discover that the products obtained by this multiplication, are the same as those obtained in the operation of division, only they occur in a reverse order. In the operation of division, each succeeding product is placed one figure farther towards the right, while in the operation of multiplication, each succeeding product is placed one figure farther towards the left. Hence the sum of the products in the case of division, must be the same as the sum in the case of multiplication. In the operation of division, by the above rule, these products are successively subtracted from the corresponding parts of the dividend, until the whole is exhausted. Now we have just shown by the operation of multiplication, that the sum of these products, taken in the order in which they stand, is equal to the dividend. Therefore the above rule for *Long Division* must be correct.

PROOF.

From what has been said, we also infer that this method *of long division* proves itself as we proceed with the work, since we have only to add the successive products, and the remainder, if any, to obtain the dividend.

What is Long Division? How do you place the numbers? Repeat the rule. If, after having brought down a new figure, the result is less than the divisor, how do you proceed? When the partial product is greater than the number which was supposed to contain the divisor, how do you do? When the remainder is greater than the divisor, how do you proceed? Explain the method of proof.

2. Divide 175678 by 223.

OPERATION.

```
223) 175678 (787
     1561  first product.
     ----
      1957
      1784  second product.
      ----
       1738
       1561  third product
       ----
        177  remainder.
        ---
```

If we take the sum of the successive products and the remainder, adding them as they now stand in the above work, we shall obtain 175678; which, agreeing with the dividend, proves the accuracy of the division. This method of proving division is perhaps as simple and brief as any method which can be devised.

The common method of proving Division, and one which is applicable to Short Division as well as to Long Division, is to multiply the divisor and quotient together, and to add in the remainder, if any.

3. Divide 7892343 by 139. *Ans.* 56779 $\frac{62}{139}$.
4. Divide 177575124270 by 753465. Ans. 235678.

5. Divide 34789205 by 64534. *Ans.* $539\frac{5379}{64534}$.
6. Divide 123456789 by 789. *Ans.* $156472\frac{381}{789}$.
7. Divide 5763447 by 678509. *Ans.* $8\frac{335375}{678509}$.
8. Divide 1521808704 by 6503456. *Ans.* 234.
9. Divide 243166625648 by 3471032.
Ans. 70056 with 7856 remainder.
10. Divide 166168212890625 by 12890625.*
Ans. 12890625.
11. Divide 11963109376 by 109376. *Ans.* 109376

CASE III.

29. When the divisor is a composite number.

We have seen (ART. **21**,) that, in multiplication, when the multiplier is a composite number, the product may be found by multiplying by the factors successively.

Now, as division is the reverse process of multiplication, it is plain that when the divisor is a composite number, the quotient may be found by dividing by the factors successively.

Divide 944 by $105 = 3 \times 5 \times 7$.

OPERATION.

1st factor 3)944
2d factor 5)314 2 = 1st rem.
3d factor 7)62 4 = 2d rem.
quotient 8 6 = 3d rem.

In this division, we find the different remainders in succession.

Let us now seek the true remainder, or that remainder which would have been found, had we at once divided the 944 by 105.

Since each unit of the 62 is 5 times as great as each unit of 314, it follows, that each unit of the 3d remainder, 6, which is a part of 62, is also 5 times as great as each unit of 314. Hence the remainder 6 is the same as 5 times 6, or 30, units of the same kind as those of 314; but the 2d remainder 4, being a part of 314, and of the same order, should be added to 30, making 34, for the true remainder arising from dividing 314 by 35 or 5×7. Again, since each unit of 314 is 3 times as great as each unit of 944, it follows, that each unit of the 34 is also 3 times as great as each unit of 944. Hence the remainder 34 is the same as 3 times $34=102$ units of the same kind as 944; but the 1st remainder, 2, being a part of 944, is of the same order; so that $102+2=104$, is the true remainder required.

---

* This question and its succeeding one are worthy of notice, since the terminal figures of the dividend, divisor, and quotient, are the same.

From the foregoing operation and reasoning, we deduce the following

RULE.

Divide the given sum by one of the factors of the divisor, and that quotient by another factor, and so on, until all the factors have been used. The last quotient will be the quotient sought. It makes no difference in what order the factors are used.

To obtain the true remainder, we must observe the following

RULE.

Multiply the last remainder by the divisor preceding the last, and add in the preceding remainder; multiply this sum by the next preceding divisor, and add in the next preceding remainder; so continue this reverse process until you have multiplied by all the divisors except the last.

How do you proceed when the divisor is a composite number? Does it make any difference which factor we first divide by? When there are several remainders, explain how the true remainder is obtained.

EXAMPLES.

1. Divide 839 by 120.

We will resolve 120 into the three factors, $4\times5\times6=120$. Now, proceding agreeably to the rule, we have the annexed operation.

OPERATION.

$$\begin{array}{r|l}
4 & \underline{839} \\
5 & \underline{209} \quad 3=\text{first rem.} \\
6 & \underline{41} \quad 4=\text{second rem.} \\
 & \underline{6} \quad 5=\text{third rem.}
\end{array}$$

Now, to obtain the true remainder, we have this

OPERATION.

$$\overset{\text{last remainder.}}{5}\times\overset{\text{div. preceding the last.}}{5}+\overset{\text{preceding remainder.}}{4}=29.\quad \text{Again, } 29\times\overset{\text{next preceding divisor.}}{4}+\overset{\text{next preceding rem.}}{3}=\overset{\text{true remainder.}}{119}.$$

Had there been more than three factors, the operation would have been equally simple, but a little more lengthy.

2. Divide 8217 by $35=5\times7$. *Ans.* 234 with 27 rem.
3. Divide 33678 by $15=3\times5$. *Ans.* 2245 with 3 rem.
4. Divide 9591 by $72=8\times9$. *Ans.* 133 with 15 rem.
5. Divide 10859 by $49=7\times7$. *Ans.* 221 with 30 rem.

CASE IV.

30. When the divisor ends with one or more ciphers.

We have seen (Art. **4**,) that a number is multiplied by 10 by annexing a cipher; it is multiplied by 100 by annexing two ciphers; by 1000 by annexing three ciphers, &c. Conversely, a number is divided by 10 by cutting off one figure from the right; it is divided by 100 by cutting off two figures from the right, &c.

EXAMPLES.

1. Divide 2475 by 20.

Having cut off the 5 from the right of the dividend, and the 0 from the right of the divisor, which is, in effect, dividing both dividend and divisor by 10, we proceed to divide 247 by 2, (Art. **25**.) We obtain 123 for a quotient and 1 for a remainder. This remainder is 1 ten, since it is a part of the 7 of the dividend which occupies the ten's place; annexing the 5 units, which was cut off, to the 1 ten which remained, we have 1 ten and 5 units, or 15 for the true remainder.

OPERATION.

2|0)247|5

123 15 remainder.

Note. This case may be comprised under Case III., Art. **29**. Thus, taking the preceding example, the divisor 20=2×10. Dividing 2475 first by 10, which division is effected by cutting off the right-hand figure, 5, we have 247 for the first quotient, and 5 for the first remainder. Next, dividing 247 by 2, we find 123 for the quotient sought, and 1 for the second remainder.

Now, by the rule under the case referred to, we find the true *remainder to be 1×10+5=15.*

2. Divide 4567894 by 3700.

OPERATION.

```
37|00)45678|94(1234
      37
      --
       86
       74
       ---
       127
       111
       ---
        168
        148
        ----
        2094=rem.
```

If in this second example, we regard the divisor 3700 as a composite number, whose factors are 100 × 37 = 3700, the example will also properly come under Case III; according to which, the 94 cut off from the right of the dividend is to be considered the *first remainder*, and the 20 is the *last remainder*. Hence, the true remainder is 20 × 100 + 94 = 2094.

From the above operations we deduce this

RULE.

Cut off from the right of the dividend as many figures as there are ciphers at the right of the divisor; divide what remains by the divisor without the ciphers at its right. To the final remainder annex the figures cut off from the dividend, for the true remainder.

How do you proceed when there are ciphers at the right of the divisor?

3. Divide 7123545 by 421000. *Ans.* 16 and 387545 rem.

4. Divide 1212121212 by 42000.
Ans. 28860 and 1212 rem.

5. Divide 123456789 by 12300. *Ans.* 10037 and 1689 rem.

6. Three men are to share equally in the sum of 1236 dollars. How many dollars will each have? *Ans.* 412 dolls.

7. Divide 1245 acres of land equally between five brothers. *Ans.* Each has 249 acres.

8. It is about 95000000 miles from here to the sun. Now, admitting that it requires 8 minutes for light to pass from the sun to the earth, how many miles does it pass in *one minute?* Ans. 11875000 miles.

9. Allowing 22 bricks to be sufficient to make one cubic foot of masonry, how many cubic feet are there in a work which requires 100000 bricks?

Ans. 4545 cubic feet and 10 brick remaining

10. The circumference of the earth is about 25000 miles. How long would it require for a person to travel around it, if he could pass uninterruptedly at the rate of 200 miles per day? *Ans.* 125 days.

11. In 1845 the extent of post-roads in the United States was 143940 miles, and the amount paid for the transportation of the mail during the same year was 2905504 dollars. How much was the average expense per mile?

Ans. Between 20 and 21 dollars.

12. The distance of Uranus from the sun is about 1860624000 miles. How many hours would it require to pass this distance at 18 miles per hour? Also, how many days, and how many years, counting 24 hours to the day, and 365 days to the year?

Ans. It would require 103368000 hours.
" " " 4307000 days.
" " " 11800 years.

13. How many barrels of apples, at 3 dollars per barrel, can I buy for 2568 dollars? And if one tree produce 8 barrels, how many trees will be required to yield the required amount?

Ans. 856 barrels.
107 trees.

31. QUESTIONS INVOLVING THE FOUR GROUND RULES.

1. A person owes to one man 375 dollars, to another he owes 708 dollars, to a third man he owes 911 dollars. How *much does he owe to* the three men? Ans. 1994 dollars.

2. A farmer has sheep in five fields; in the first, he has 917; in the second, 249; in the third, 413; in the fourth, 1000; and in the fifth, he has 197. How many sheep has he in the five fields? *Ans.* 2776 sheep.

3. A person owes to one man 302 dollars, to another man he owes 707 dollars, and has owing to him 2000 dollars. How much will remain after paying his debts?

Ans. 991 dollars.

4. A farmer receives for his wheat 103 dollars, for his corn 60 dollars, for his butter 511 dollars, for his cheese 1212 dollars, for his pork 601 dollars. He pays towards a new farm 1000 dollars, for a new wagon 50 dollars, for hired help on his farm 290 dollars, for repairing house 173 dollars. How much money has he remaining?

Ans. 974 dollars.

5. A person wills 1200 dollars to his wife, 300 dollars for charitable purposes, and what remains is to be equally divided among 6 children. Allowing his property to amount to 8562 dollars, how much would each child have?

Ans. 1177 dollars.

6. A man gave 13558 dollars for a farm; he then sold 73 acres, at 75 dollars per acre; the remainder stood him in at 59 dollars per acre. How many acres did he purchase?

Ans. 210 acres.

7. Four boys divide 336 apples as follows: the first takes one sixth of the whole; the second takes one fourth of what was left; the third takes one half of what was then left; the fourth has the remainder. What number of apples did each boy have?

Ans. The first had 56.
The second had 70.
The third had 105.
The fourth had 105.

8. An estate of 8100 dollars was divided among 9 children in the following way: the first had 100 dollars and one tenth of the remainder; after this the second had 200 dollars and one tenth of the residue; again, the third had 300 dollars and one tenth of the remainder, and so on; each succeeding child had 100 dollars more than the one immediately preceding, and then one tenth of what still remained. What was the share of each?

Ans. { They shared equal; each had 900 dollars.

9. A and B each owe C: A owes 1472 dollars, which is less than what B owes him, and yet the difference between A's and B's debts is 719 dollars. How much does B owe C? *Ans.* 2191 dollars.

10. Admitting the earth to move 68000 miles per hour, how far will it move in one day; and how far in a year of 365 days? *Ans.* { 1632000 miles in one day. 595680000 miles in one year.

11. If the President of the United States expends daily 60 dollars, how much will he be able to save at the end of the 365, out of his salary of 25000 dollars?

Ans. 3100 dollars.

12. An army, consisting of 4525 men, have 103075 loaves of bread. At the end of 21 days, 500 men are killed in a battle. Now, if each man in each day eat one loaf of bread, how many days after the battle will the bread sustain the army? *Ans.* 2 days.

13. Two locomotives start from the same place, and move in the same direction; the first goes 25 miles each hour, the second only 15 miles. After the first has passed a distance of 100 miles it commences a backward motion, maintaining the *same* velocity, until it meets the second

locomotive. How many hours after starting will they meet? And at what distance will they meet from the starting point? *Ans.* { They will meet in 5 hours, at a distance of 75 miles.

14. One hundred miles of railroad track are to be laid with heavy rail, requiring 116 tons to the mile. After receiving iron at 52 dollars per ton to lay 58 miles, the price per ton was increased so as to make the whole cost of the entire road 612944 dollars. What was the latter price per ton of the iron? *Ans.* 54 dollars.

FRACTIONS.

32. A fraction is a part of a unit.

Several methods are used to express fractions or parts of units, which give rise to several distinct kinds of fractions. Those usually employed in arithmetic are VULGAR OR COMMON FRACTIONS, and DECIMAL FRACTIONS.

What is a fraction? What two methods are usually employed to express fractions?

VULGAR FRACTIONS.

33. Vulgar fractions consist of two distinct parts or terms, the one written above the other, with a straight horizontal line between them, as in division, (ART. **23.**) The number above the line is called the *numerator*. The number below the line is called the *denominator*. The denominator shows how many parts the unit is divided into; and the numerator shows how many parts are used.

Thus $\frac{5}{8}$ is a vulgar fraction, whose numerator is 5 and denominator 8: it is read *five eighths.*

A vulgar fraction may be considered a concise method of expressing division, (ART. **23**,) where the numerator corresponds to the dividend, and the denominator to the divisor. Thus $\frac{5}{8}$ is the same as 5 divided by 8, and it may therefore be read *one eighth of five*, or, as above, *five eighths of one.* In the same way $\frac{1}{9}$ indicates that 1 is divided into 9 equal parts: it is read *one ninth of one.* After the same manner,

$\frac{3}{7}$ is read *one seventh of three*, or *three sevenths of one.*
$\frac{4}{5}$ is read *one fifth of four*, or *four fifths of one.*
$\frac{6}{11}$ is read *one eleventh of six*, or *six elevenths of one.*
$\frac{8}{9}$ is read *one ninth of eight*, or *eight ninths of one.*
&c. &c. &c.

The fraction	$\frac{5}{7}$	denotes that	5	is to be divided by	7.
"	$\frac{13}{4}$	"	13	"	4.
"	$\frac{17}{8}$	"	17	"	8.
"	$\frac{3}{12}$	"	3	"	12.
"	$\frac{1}{2}$	"	1	"	2.
"	$\frac{1}{3}$	"	1	"	3.
"	$\frac{1}{4}$	"	1	"	4.
"	$\frac{2}{5}$	"	2	"	5.
	&c.		&c.		&c.

When the numerator is equal to the denominator, the value of the fraction is a unit.

When the numerator is less than the denominator, the value is less than a unit, and the expression is called a *proper fraction.*

When the numerator is greater than the denominator, the value is greater than a unit, and the expression is called an *improper fraction.*

Thus, each of the expressions, $\frac{3}{3}$, $\frac{5}{5}$, $\frac{10}{10}$, $\frac{13}{13}$, &c., is equal to a unit.

Each of the expressions, $\frac{1}{2}$, $\frac{2}{7}$, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{6}{7}$, $\frac{1}{10}$, &c., is a proper fraction.

Each of the expressions, $\frac{9}{1}$, $\frac{7}{5}$, $\frac{8}{6}$, $\frac{12}{11}$, $\frac{9}{2}$, $\frac{17}{13}$, &c., is an improper fraction.

When a whole number and fraction are connected, the expression is called a *mixed number.* Thus, $4\frac{1}{2}$, $3\frac{1}{7}$, $5\frac{1}{17}$, $2\frac{20}{27}$, &c., are mixed numbers. The whole number is called the *integral part* of the expression, and the fraction is called the *fractional* part.

When several fractions are connected by the word *of,* the expression is called a *compound fraction.* The expressions, $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{9}$, $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{8}{5}$ of $\frac{11}{10}$, $\frac{3}{7}$ of $\frac{3}{8}$ of $\frac{2}{9}$ of $\frac{5}{6}$, &c., are compound fractions.

Any number may be made to assume the form of an improper fraction, by writing under it a unit for the denominator. Thus, 2, 3, 4, 5, 7, &c., are the same as $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$, $\frac{5}{1}$, $\frac{7}{1}$, &c.

Fractions sometimes occur, in which the numerator, or denominator, or both, are themselves fractional; such expressions are called *complex fractions.*

Thus, $\dfrac{3\frac{1}{2}}{4}$, $\dfrac{4}{7\frac{1}{3}}$, $\dfrac{2\frac{1}{5}}{3\frac{1}{7}}$, $\dfrac{10\frac{1}{9}}{9\frac{1}{10}}$, &c., are complex fractions.

A fraction is said to be inverted when the numerator and denominator exchange places. Thus: the fractions, $\frac{2}{3}$, $\frac{5}{4}$, $\frac{7}{8}$, $\frac{6}{10}$, $\frac{4}{9}$, $\frac{3}{7}$, when inverted, become $\frac{3}{2}$, $\frac{4}{5}$, $\frac{8}{7}$, $\frac{10}{6}$, $\frac{9}{4}$, $\frac{7}{3}$.

What is a vulgar fraction? Which is the numerator of a vulgar fraction? Which the denominator? What does the denominator show? What does the numerator show? *In the* vulgar fraction five eighths, which is the numerator, and which the *denominator?* How is it read? What may a vulgar fraction be considered a concise

way of expressing ? In a vulgar fraction, which part corresponds to the dividend, and which to the divisor ? What is the value of the fraction, when the numerator is equal to the denominator ? When is the value less than a unit ? What is the fraction then called ? When is the value greater than a unit ? What is the fraction then called ? Give examples of proper fractions. Give examples of improper fractions. When a whole number and fraction are connected, what is the expression called ? Give examples. When several fractions are connected by the word *of*, what kind of a fraction is it then called ? Give examples. When the numerator, or denominator, or both, are already fractional, what are they called ? Give examples. When is a fraction said to be inverted ? Give examples.

REDUCTION OF FRACTIONS.

34. In division, the *divisor*, *dividend* and *quotient* are so related, *that the product of the divisor and quotient is always equal to the dividend.* Hence, the divisor and quotient may be interchanged; that is, if the dividend be divided by the quotient, the result will be the divisor. It is also obvious, that, with the same divisor, twice as great a dividend will give twice as great a quotient; thrice as great a dividend will give thrice as great a quotient; and in general, the effect of multiplying the dividend by any number is to multiply the quotient by the same number. On the other hand, if the dividend remain the same, multiplying the divisor by any number produces the same effect as dividing the quotient by the same number. Consequently, if we multiply both dividend and divisor by the same number, it will produce no change in the quotient.

Again, it is obvious, that with the same divisor, half as great a dividend will give but half as great a quotient; one-third as great a dividend will give one-third as great a quotient; and in general, the effect of dividing the dividend by any number, is to divide the quotient by the same number. On the other hand, if the dividend remain the same, dividing the divisor by any number produces the same effect as multiplying the quotient by the same number. Consequently, if we divide both dividend and divisor by the same number, it will produce no change in the quotient.

If, now, we call to mind that the value of a fraction is the quotient arising from dividing the numerator by the denominator, we readily infer the following

PROPOSITIONS.

I. That, multiplying the numerator by any number is the same as multiplying the value of the fraction by the same number.

II. That, multiplying the denominator by any number is the same as dividing the value of the fraction by the same number.

III. That, multiplying both numerator and denominator by any number does not alter the value of the fraction.

IV. That, dividing the numerator by any number is the same as dividing the value of the fraction by the same number.

V. That, dividing the denominator by any number is the same as multiplying the value of the fraction by the same number.

VI. That, dividing both numerator and denominator by the same number does not alter the value of the fraction.

GREATEST COMMON DIVISOR.

35. The greatest common divisor of two or more numbers, is the greatest number which will divide them *without* any remainder.

Before proceeding to find the *greatest common* divisor of two numbers, we will show that any number which will divide two numbers exactly, will also divide their difference.

Suppose we have a common divisor of 636 and 276; this will also exactly divide 360, their difference. For, 636 is made up of the two parts 276 and 360, so that any number which will exactly divide 636, will also divide 276+360; if a divisor of 636 will at the same time divide one of its parts, 276, it will of necessity divide the other part, 360. Hence a common divisor of 636 and 276 is also a divisor of their difference, 360.

As the divisor which is common to 636 and 276, is also a divisor of 360, it must be a common divisor of 360 and 276, and consequently of 84, the difference between 360 and 276; and in general, when any two numbers have a common divisor, and we subtract any number of times the smaller number from the larger, the remainder will be exactly divisible by this common divisor.

What, now, is the greatest common divisor of 360 and 276.

The greatest divisor cannot exceed the less number, 276. But 276 will not divide the other number, 360, without a remainder, 84. Hence, the greatest divisor of 276 and 84 must be the greatest common divisor of 360 and 276. Again, dividing 276 by 84, we find 3, quotient, and 24, remainder. So the greatest common divisor of 84 and 24 is also the greatest common divisor of 276 and 84, and consequently of 360 and 276. Now, dividing 84 by 24, we find the quotient 3, and remainder 12. Finally, dividing 24 by 12, we find it is contained exactly twice; so that the greatest common divisor of 24 and 12 is 12: consequently, *12 is the greatest* common divisor of 360 and *276. We will exhibit* in one point of view the above.

OPERATION.

276)360(1
276

84)276(3
252

24)84(3
72

12)24(2
24

0

Hence, to find the greatest common divisor of two numbers, we deduce this

RULE.

Divide the greater number by the less, then the less number by the remainder; thus continue to divide the last divisor by the last remainder, until there is no remainder. The last divisor will be the greatest common divisor.

NOTE.—When there are more than two numbers whose greatest common divisor is required, we must find the greatest common divisor of any two, and then find the greatest common divisor of this divisor thus found, and one of the remaining numbers; and thus continue until all the different numbers have been used.

What is the greatest common divisor of two or more numbers? Repeat the rule for finding the greatest common divisor of two numbers. How do you proceed when there are more than two numbers?

EXAMPLES.

1. Find the greatest common divisor of 592 and 999

OPERATION.

```
592)999(1
    592
    ---
    407)592(1
        407
        ---
        185)407(2
            370
            ---
             37)185(5
                185
                ---
                  0
```

From which we obtain 37 for the greatest common divisor of 592 and 999.

2. What is the greatest common divisor of 492, 744, and 906?

We first find the greatest common divisor of 492 and 744 by the following

OPERATION.

```
492)744(1
    492
    ---
    252)492(1
        252
        ---
        240)252(1
            240
            ---
             12)240(20
                240
                ---
                  0
```

Therefore, the greatest common divisor of 492 and 744 is 12.

Again, proceeding with 12 and 906, we have the following

OPERATION.

```
12)906(75
   900
   ------
     6)12(2
       12
       --
        0
```

We thus find 6 to be the greatest common divisor of 12 and 906, and consequently of the three numbers, 492, 744, and 906.

3. What is the greatest common divisor of 315 and 405? *Ans.* 45.

4. What is the greatest common divisor of 1825 and 2655? *Ans.* 5.

5. What is the greatest common divisor of 506 and 308? *Ans.* 22.

6. What is the greatest common divisor of 404 and 364? *Ans.* 4.

7. What is the greatest common divisor of 246, 372, and 522? *Ans.* 6.

36. We are now prepared to proceed to the reduction of fractions.

We know (PROP. VI., ART. **34**) that we can divide both numerator and denominator of a fraction by any number without altering its value. If we divide by the greatest common divisor, the resulting fraction will be in its *lowest terms.*

Therefore, to reduce a fraction to its lowest terms, we have this

RULE.

Divide both numerator and denominator by their greatest common divisor.

How do you reduce a fraction to its lowest terms?

EXAMPLES.

1. Reduce $\frac{592}{999}$ to its lowest terms.

We have already found (Ex. 1, ART. **35**,) the greatest common divisor of 592 and 999 to be 37. Dividing both these terms by 37, we find 16 and 27 for quotients: hence, $\frac{592}{999}$, when reduced to its lowest terms, becomes $\frac{16}{27}$.

2. Reduce $\frac{1074}{8592}$ to its lowest terms. *Ans.* $\frac{1}{8}$.

3. Reduce $\frac{30}{60}$, $\frac{60}{180}$, $\frac{45}{180}$, to their lowest terms. *Ans.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

4. Reduce $\frac{315}{405}$ to its lowest terms. *Ans.* $\frac{7}{9}$.

5. Reduce $\frac{172}{1118}$ to its lowest terms. *Ans.* $\frac{2}{13}$.

6. Reduce $\frac{275}{440}$ to its lowest terms. *Ans.* $\frac{5}{8}$.

7. Reduce $\frac{99}{9999}$ to its lowest terms. *Ans.* $\frac{1}{101}$.

8. Reduce $\frac{4104}{4560}$ to its lowest terms. *Ans.* $\frac{9}{10}$.

9. Reduce $\frac{33522}{41223}$ to its lowest terms. *Ans.* $\frac{74}{91}$.

10. Reduce $\frac{58760}{66105}$ to its lowest terms. *Ans.* $\frac{8}{9}$.

We may frequently discover numbers, by inspection, which will divide both numerator and denominator without a remainder. When this is the case, we need not resort to the rule for obtaining the greatest common divisor, until we have divided by such numbers.

11. Reduce $\frac{5184}{6912}$ to its lowest terms.

In this example, we first divide the numerator and denominator by 4, which reduces the fraction to $\frac{1296}{1728}$. We again divide by 4, and obtain $\frac{324}{432}$. Dividing the numerator and denominator of this last fraction by 4, we obtain $\frac{81}{108}$, which is still further reduced by dividing three successive times by 3.

$$\frac{5184}{6912} \overset{\div 4}{=} \frac{1296}{1728} \overset{\div 4}{=} \frac{324}{432} \overset{\div 4}{=} \frac{81}{108} \overset{\div 3}{=} \frac{27}{36} \overset{\div 3}{=} \frac{9}{12} \overset{\div 3.}{=} \frac{3}{4}.$$

12. Reduce $\frac{162}{324}$ to its lowest terms.

$$\frac{162}{324} \overset{\div 2}{=} \frac{81}{162} \overset{\div 3}{=} \frac{27}{54} \overset{\div 3}{=} \frac{9}{18} \overset{\div 3}{=} \frac{3}{6} \overset{\div 3.}{=} \frac{1}{2}.$$

13. Reduce $\frac{360}{420}$ to its lowest terms. *Ans.* $\frac{6}{7}$.

14. Reduce $\frac{1386}{1638}$ to its lowest terms. *Ans.* $\frac{11}{13}$.

15. Reduce $\frac{14175}{22050}$ to its lowest terms. *Ans.* $\frac{9}{14}$.

16. Reduce $\frac{472500}{787500}$ to its lowest terms. *Ans.* $\frac{3}{5}$.

37. To reduce an improper fraction to a whole or mixed number.

Reduce $\frac{95}{13}$ to a mixed number.

Since the value of a fraction is the quotient arising from dividing the numerator by the denominator, (ART. **34**,) we may find the value of $\frac{95}{13}$, by dividing 95 by 13. Performing the division, we find 7 for the quotient, and 4 for a remainder. Hence $\frac{95}{13}=7\frac{4}{13}$. (ART. **26**.)

From which we have the following

RULE.

Divide the numerator by the denominator; the quotient will be the integral part of the mixed number. The remainder being placed over the denominator of the improper fraction, will form the fractional part.

Repeat the rule for reducing an improper fraction to a mixed number.

EXAMPLES.

1. Reduce $\frac{60}{25}$ to a mixed number. *Ans.* $2\frac{2}{5}$.
2. Reduce $\frac{80}{7}$, $\frac{90}{12}$, to mixed numbers. *Ans.* $11\frac{3}{7}$, $7\frac{1}{2}$.
3. Reduce $\frac{50}{6}$, $\frac{80}{9}$, to mixed numbers. *Ans.* $8\frac{1}{3}$, $8\frac{8}{9}$.
4. Reduce $\frac{131}{7}$ to a mixed number. *Ans.* $18\frac{5}{7}$.
5. Reduce $\frac{29817}{365}$ to a mixed number. *Ans.* $81\frac{252}{365}$.
6. Reduce $\frac{3307}{97}$ to a mixed number. *Ans.* $34\frac{9}{97}$.
7. Reduce $\frac{3313}{101}$ to a mixed number. *Ans.* $32\frac{81}{101}$.
8. Reduce $\frac{3407}{89}$ to a mixed number. *Ans.* $38\frac{25}{89}$.
9. Reduce $\frac{33045}{3203}$ to a mixed number. *Ans.* $10\frac{1015}{3203}$.

38. To reduce a mixed number to an improper fraction.

Reduce the mixed number $37\frac{3}{8}$ to an improper fraction. If we multiply the fractional part, $\frac{3}{8}$, by 8, the product will be 3. (ART. **34.**) Multiplying 37 by 8 we obtain 296, to which adding 3, we find 299 for 8 times $37\frac{3}{8}$. Hence $37\frac{3}{8}$ is equal to 299 divided by 8, that is, to $\frac{299}{8}$.

Hence, we have this

RULE.

Multiply the integral part of the mixed number by the denominator of the fractional part; to the product add the numerator of the fractional part; the sum will be the numerator of the improper fraction; under which place the denominator of the fractional part.

This rule is obviously correct, since it is the reverse of the rule, (ART. **37**,) where a reverse operation was required to be performed.

EXAMPLES.

1. Reduce $4\frac{1}{2}$ to an improper fraction. *Ans.* $\frac{9}{2}$.
2. Reduce $3\frac{1}{3}$, $7\frac{2}{5}$, to improper fractions. *Ans.* $\frac{10}{3}$, $\frac{37}{5}$.
3. Reduce $8\frac{1}{3}$, $7\frac{1}{2}$, to improper fractions. *Ans.* $\frac{25}{3}$, $\frac{15}{2}$

4. Reduce $81\frac{252}{365}$ to an improper fraction. *Ans.* $\frac{29817}{365}$.
5. Reduce $37\frac{37}{41}$ to an improper fraction. *Ans.* $\frac{1554}{41}$.
6. Reduce $3\frac{321}{331}$ to an improper fraction. *Ans.* $\frac{1314}{331}$.
7. Reduce $7\frac{17}{19}$ to an improper fraction. *Ans.* $\frac{150}{19}$.
8. Reduce $365\frac{81}{83}$ to an improper fraction. *Ans.* $\frac{30376}{83}$.
9. Reduce $1234\frac{29}{31}$ to an improper fraction. *Ans.* $\frac{38283}{31}$.
10. Reduce $77\frac{7}{11}$ to an improper fraction. *Ans.* $\frac{854}{11}$.

39. Let us endeavor to reduce the compound fraction $\frac{3}{4}$ of $\frac{7}{11}$ to an equivalent simple fraction.

$\frac{1}{4}$ of $\frac{7}{11}$ can be obtained by dividing the value of the fraction $\frac{7}{11}$ by 4, which (by PROP. II., ART. **34**,) can be effected by multiplying the denominator by 4; therefore, $\frac{1}{4}$ of $\frac{7}{11}$ equals $\frac{7}{4\times 11}$.

Again, $\frac{3}{4}$ of $\frac{7}{11}$ is obviously three times as great as $\frac{1}{4}$ of $\frac{7}{11}$; therefore, to obtain $\frac{3}{4}$ of $\frac{7}{11}$, we must multiply $\frac{7}{4\times 11}$ by 3, which (by PROP. I., ART. **34**,) can be done by multiplying the numerator by 3; hence we have $\frac{3}{4}$ of $\frac{7}{11} = \frac{3\times 7}{4\times 11} = \frac{21}{44}$.

Therefore, to reduce compound fractions to their equivalent simple ones, we have this

RULE.

Consider the word OF, *which connects the fractional parts, as equivalent to the sign of multiplication. Then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; always* *observing to reject or cancel such factors as are common to the numerators and denominators, which is the same as dividing both numerator and denominator by the same numbers, and which (by* PROP. VI, ART. **34**,) *does not change the value of the resulting fraction.*

Repeat the Rule for reducing a compound fraction to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{15}$ of $\frac{5}{12}$ to its equivalent simple fraction.

Substituting the sign of multiplication for the word *of* we get $\frac{1}{2}\times\frac{3}{4}\times\frac{8}{15}\times\frac{5}{12}$. First, cancelling the 8 of the numerator against the 2 and 4 of the denominator, by drawing a line across them, we get

$$\frac{1}{\cancel{2}}\times\frac{3}{\cancel{4}}\times\frac{\cancel{8}}{15}\times\frac{5}{12}.$$

Again, cancelling the 3 and 5 of the numerator against the 15 of the denominator, we finally obtain

$$\frac{1}{\cancel{2}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{8}}{\cancel{15}}\times\frac{\cancel{5}}{12}=\frac{1}{12}.$$

2. Reduce $\frac{3}{7}$ of $\frac{14}{35}$ of $\frac{7}{8}$ of $\frac{4}{9}$ of $\frac{5}{11}$ to its simplest form.

First, cancelling the 7 and 5 of the numerator against the 35 of the denominator, we get

$$\frac{3}{7}\times\frac{14}{\cancel{35}}\times\frac{\cancel{7}}{8}\times\frac{4}{9}\times\frac{\cancel{5}}{11}.$$

Again, cancelling the 7 of the denominator against the same factor of the 14 of the numerator, and the 3 of the numerator against the same factor of the 9 of the denominator, we obtain

$$\frac{\cancel{3}}{\cancel{7}}\times\frac{\overset{2}{\cancel{14}}}{\cancel{35}}\times\frac{\cancel{7}}{8}\times\frac{4}{\cancel{9}}\times\frac{\cancel{5}}{11}.$$

Finally, cancelling the 2 and 4 of the numerator against e 8 of the denominator, we get

$$\frac{\cancel{3}}{\cancel{7}}\times\frac{\overset{\cancel{2}}{\cancel{14}}}{\cancel{35}}\times\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{4}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{5}}{11}=\frac{1}{33}.$$

NOTE.—We have written our fractions several times, in order more clearly to exhibit the process of cancelling. But in prac- e, it will not be necessary to write the fractions more than once. will make no difference which of the factors is first cancelled. hen all the common factors have in this way been stricken out, fraction will then appear in its lowest terms.

3. Reduce $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{7}{8}$ to its simplest form.
Ans. $\frac{1}{8}$.

4. Reduce $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{1}{7}$ to its simplest form.
Ans. $\frac{1}{28}$.

5. Reduce $\frac{3}{7}$ of $\frac{14}{15}$ of $\frac{10}{12}$ of $\frac{3}{7}$ of $\frac{14}{17}$ to its simplest form.
Ans. $\frac{2}{17}$.

6. Reduce $\frac{1}{2}$ of $2\frac{1}{2}$ of $3\frac{1}{3}$ of 6 to its simplest form.
Ans. 25.

7. Reduce $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{3}{8}$ of $\frac{7}{15}$ to its simplest form.
Ans. $\frac{1}{40}$.

8. Reduce $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{14}{21}$ of $\frac{42}{60}$ to its simplest form.
Ans. $\frac{1}{15}$.

9. Reduce $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{10}{35}$ of $\frac{7}{8}$ of $\frac{5}{6}$ of $4\frac{1}{2}$ to its simplest rm.
Ans. $\frac{3}{16}$.

.0. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ to simplest form.
Ans. $\frac{1}{10}$.

40. To reduce fractions to a common denominator

We know (ART. 34, PROP. III.,) that the value of a fraction is not changed by multiplying both numerator and denominator by the same number. If, then, we multiply the numerator and denominator of each fraction by the product of the denominators of all the other fractions, we shall retain the values of the respective fractions, and at the same time they will have a common denominator.

Let it be required to reduce $\frac{1}{2}$, $\frac{5}{3}$ of $\frac{2}{5}$, $\frac{3}{11}$, and $\frac{7}{9}$ of $\frac{2}{7}$, to equivalent fractions having a common denominator.

These fractions, when reduced to their simplest form, become $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{11}$, and $\frac{2}{9}$.

For first fraction, $\frac{1}{2}$.

Multiply the numerator and denominator, each by $3\times 11\times 9$, the product of the denominators of the other fractions, and we find

$1\times 3\times 11\times 9=297$ for new numerator.
$2\times 3\times 11\times 9=594$ " " denominator.

For second fraction, $\frac{2}{3}$.

Multiply the numerator and denominator, each by $2\times 11\times 9$, the product of the denominators of the other fractions, and we find

$2\times 2\times 11\times 9=396$ for new numerator.
$3\times 2\times 11\times 9=594$ " " denominator.

For third fraction, $\frac{3}{11}$.

Multiply the numerator and denominator, each by $2\times 3\times 9$, the product of the denominators of the other fractions, and we find

$3\times 2\times 3\times 9=162$ for new numerator.
$11\times 2\times 3\times 9=594$ " " denominator.

For fourth fraction, $\frac{2}{9}$.

Multiply the numerator and denominator, each by $2\times$

3×11, the product of the denominators of the other fractions, and we find

$2 \times 2 \times 3 \times 11 = 132$ for new numerator.

$9 \times 2 \times 3 \times 11 = 594$ " " denominator.

Hence the fractions become $\frac{297}{594}$, $\frac{396}{594}$, $\frac{162}{594}$, and $\frac{132}{594}$.

It will be seen that each numerator is multiplied by the product of all the denominators except its own. It will also be seen that in obtaining each new denominator, the factors are the same, namely, all the denominators.

Hence the following

RULE.

Reduce mixed numbers to improper fractions, and compound fractions to their simplest form. Then multiply each numerator by all the denominators except its own for a new numerator, and all the denominators together for a common denominator.

Repeat this Rule.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$ to equivalent fractions having a common denominator. *Ans.* $\frac{35}{105}$, $\frac{21}{105}$, $\frac{15}{105}$.

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, to equivalent fractions having a common denominator. *Ans.* $\frac{12}{24}$, $\frac{8}{24}$, $\frac{6}{24}$.

3. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, to equivalent fractions having a common denominator. *Ans.* $\frac{240}{360}$, $\frac{270}{360}$, $\frac{288}{360}$, $\frac{300}{360}$.

4. Reduce $\frac{1}{2}$ of $\frac{2}{3}$, $4\frac{1}{2}$, $5\frac{1}{3}$, to equivalent fractions having a common denominator. *Ans.* $\frac{6}{18}$, $\frac{81}{18}$, $\frac{96}{18}$.

5. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of 5, $7\frac{1}{3}$, $5\frac{1}{5}$, to equivalent fractions having a common denominator. *Ans.* $\frac{75}{30}$, $\frac{220}{30}$, $\frac{156}{30}$.

41. In most cases fractions may be reduced to equivalent ones having a smaller common denominator than is given by the above rule. Before showing how to find the *least common denominator* of fractions, it becomes necessary to show how to find

THE LEAST COMMON MULTIPLE.

A *multiple* of several numbers is such a number as can be divided by each of them without a remainder. Thus, 12, 24, 36, 48, &c., are multiples of 2, 3, 4, and 6, since each of them is divisible by 2, 3, 4, and 6. Any set of numbers may have an infinite number of multiples. In practice it is the *least* common multiple which is usually sought. In the above example, 12 is the least common multiple of 2, 3, 4, and 6.

Let us seek the least common multiple of two numbers, as for example, of 4 and 18. Separating these numbers into their smallest component parts, (ART. **21**,) they become $4=2\times2$; $18=2\times3\times3$. If we multiply $2\times2=4$ by $2\times3\times3=18$, we shall obtain $2\times2\times2\times3\times3$, which is obviously a common multiple of 4 and 18, since the factors of these numbers are found in this expression. But it is not the *least common multiple* of 4 and 18, since one of the 2's, which is a common factor of 4 and 18, may be omitted, and the result, $2\times2\times3\times3$, will still contain all the different factors of 4 and 18. Hence, when two numbers have no common divisor, their least common multiple may be found by taking their product. When they have a common divisor, their least common multiple may be found by dividing their product by their greatest common divisor; or, by dividing one of the numbers by their greatest common divisor, and multiplying the quotient by the other number; or, by dividing each number by their greatest common divisor, and multiplying the *product of the quotients* by this greatest common divisor.

It is the last method that we find most convenient to employ.

The least common multiple of more than two numbers may be found, by first finding the least common multiple of any two of the numbers, and then finding the least common multiple of that multiple, and another of the given numbers, and so on, until all the different numbers have been used.

We will now seek the least common multiple of 10, 18 and 21.

The greatest common divisor of 10 and 18 is 2. Dividing 10 and 18 by 2, we find 5 and 9 for the quotients; hence the least common multiple of 10 and 18 is $2\times5\times9$. We now seek the least common multiple of $2\times5\times9$ and 21. The greatest common divisor of these two numbers is 3, it being a divisor of 9 and of 21.

Dividing by 3, we have $2\times5\times3$ and 7 for the quotients. Hence the least common multiple of $2\times5\times9=90$ and 21, is $3\times2\times5\times3\times7=630$, which is also the least common multiple of 10, 18 and 21.

$$\begin{array}{r|r}
2 & 10,\ 18,\ 21 \\ \hline
3 & 5,\ \ 9,\ 21 \\ \hline
 & 5,\ \ 3,\ \ 7
\end{array}$$

$$2\times3\times5\times3\times7=630.$$

If we place the numbers 10, 18 and 21 in a horizontal line, and divide the 10 and 18 by 2, and bring down the 21, we shall obtain a second line consisting of 5, 9, and 21. Dividing the 9 and 21 of this second line by 3, we obtain a third line consisting of 5, 3, and 7, no two of which have a common divisor. Now, multiplying the divisors 2 and 3 by the product of the numbers in the last horizontal line, we have 630, the least *common multiple* sought.

Hence the least common multiple of any set of numbers may be found by the following

RULE.

Write the numbers in a horizontal line; divide them by the least number which will divide two or more of them without a remainder; place the quotients with the undivided numbers, if any, for a second horizontal line; proceed with this second line as with the first; and so continue until there are no two numbers which can be exactly divided by the same divisor. The continued product of the divisors, and of the numbers in the last horizontal line, will give the least common multiple.

Note.—When there is no number which will divide two of the given numbers, their continued product must be taken for the least common multiple.

What is a multiple of several numbers? Mention some of the multiples of 2, 3, 4, and 6. Are the number of multiples of any set of numbers limited? Repeat the Rule for finding the least common multiple of any set of numbers. When there is no number which will divide two of the given numbers, how is the least multiple found?

EXAMPLES.

1. What is the least common multiple of 12, 16, and 24?

OPERATION.

$$\begin{array}{r|rrr}
2 & 12, & 16, & 24. \\ \hline
2 & 6, & 8, & 12. \\ \hline
2 & 3, & 4, & 6. \\ \hline
3 & 3, & 2, & 3. \\ \hline
 & 1, & 2, & 1. \\ \hline
\end{array}$$

Hence, $2\times2\times2\times3\times2=48$ is the least common multiple.

2. What is the least common multiple of 12, 15, 24?

OPERATION.

$$\begin{array}{r|rrr} 2 & 12, & 15, & 24. \\ \hline 2 & 6, & 15, & 12. \\ \hline 3 & 3, & 15, & 6. \\ \hline & 1, & 5, & 2. \end{array}$$

Therefore, $2\times2\times3\times5\times2=120$ is the multiple sought

3. What is the least common multiple of 11, 77, 88?
Ans. 616.

4. What is the least common multiple of 37, 41?
Ans. 1517.

5. What is the least common multiple of 24, 60, 45, 180?
Ans. 360.

6. What is the least common multiple of 2, 4, 6, 8?
Ans, 24.

7. What is the least common multiple of 3, 5, 7, 9?
Ans. 315.

8. What is the least common multiple of 2, 3, 4, 5, 6, 7, 8, 9?
Ans. 2520.

9. What is the least common multiple of 7, 14, 16, 18, 24?
Ans. 1008.

10. What is the least common multiple of 1, 2, 3, 4, 5, 6, 7, 8, 9, 11?
Ans. 27720.

42. We are now prepared to reduce fractions to their *least common* denominator.

Let it be required to reduce to the least common denominator the fractions $\frac{5}{12}$, $\frac{7}{16}$, and $\frac{11}{24}$.

If we take the least common multiple of the denominators 12, 16, and 24, which is 48, and divide it in turn by these denominators, we shall obtain the respective quotients 4, 3, and 2. Hence, if we multiply the numerator and denominator of each fraction by 4, 3 and 2 respectively, they will become $\frac{20}{48}$, $\frac{21}{48}$ and $\frac{22}{48}$. These fractions are equivalent to the original ones, and have their least common denominator. Hence fractions may be reduced to their least common denominator by the following

RULE.

Reduce the fractions to their simplest form; then find the least common multiple of their denominators, (by Rule under ART. **41**,) *which will be their least common denominator. Divide this denominator by the respective denominators of the given fractions; multiply the quotients thus obtained by the respective numerators, and the several products will be the new numerators.*

Repeat the Rule for reducing fractions to their least common denominator.

EXAMPLES.

1. Reduce $\frac{5}{12}$, $\frac{7}{15}$, $\frac{11}{24}$, to equivalent fractions having the least common denominator.

The least common multiple of the denominators 12, 15 24, is 120 = common denominator.

New numerator of first fraction $\frac{120}{12} \times 5 = 50$.

New numerator of second fraction $\frac{120}{15} \times 7 = 56$.

New numerator of third fraction $\frac{120}{24} \times 11 = 55$.

Hence, the fractions, when reduced to their least common denominator, become

$$\frac{50}{120}, \frac{56}{120}, \frac{55}{120}.$$

2. Reduce $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{7}{12}$, $\frac{3}{20}$, $\frac{7}{15}$, to equivalent fractions having the least common denominator.

Ans. $\frac{15}{120}$, $\frac{18}{120}$, $\frac{56}{120}$.

3. Reduce $3\frac{1}{2}$, $4\frac{1}{3}$, $\frac{6}{5}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{105}{30}$, $\frac{130}{30}$, $\frac{36}{30}$.

4. Reduce $\frac{8}{9}$, $\frac{7}{15}$, $\frac{13}{20}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{160}{180}$, $\frac{84}{180}$, $\frac{117}{180}$.

5. Reduce $\frac{4}{15}$, $\frac{5}{11}$, $6\frac{3}{22}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{88}{330}$, $\frac{150}{330}$, $\frac{2025}{330}$.

6. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $3\frac{1}{4}$, and $\frac{1}{5}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{195}{60}$, $\frac{12}{60}$.

7. Reduce $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{7}$, $\frac{4}{21}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{21}{210}$, $\frac{70}{210}$, $\frac{30}{210}$, $\frac{40}{210}$.

8. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{20}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$, $\frac{27}{60}$.

9. Reduce $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{9}{10}$, $\frac{7}{120}$, to equivalent fractions having the least common denominator.

Ans. $\frac{80}{120}$, $\frac{30}{120}$, $\frac{72}{120}$, $\frac{100}{120}$, $\frac{108}{120}$, $\frac{7}{120}$.

10. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, to equivalent fractions having the least common denominator.

Ans. $\frac{1260}{2520}$, $\frac{840}{2520}$, $\frac{630}{2520}$, $\frac{504}{2520}$, $\frac{420}{2520}$, $\frac{360}{2520}$, $\frac{315}{2520}$, $\frac{280}{2520}$.

ADDITION OF FRACTIONS.

43. Suppose we wish to add $\frac{3}{7}$ and $\frac{1}{5}$. We know that *so long as* these fractions have different denominators, they cannot be added any more than pounds and yards can be added together. We will therefore reduce them to a common denominator. We thus obtain

$$\tfrac{3}{7}=\tfrac{15}{35},\ \tfrac{4}{5}=\tfrac{28}{35}.$$

Now, taking their sum, we obtain

$$\frac{3}{7}+\frac{4}{5}=\frac{15}{35}+\frac{28}{35}=\frac{15+28}{35}=\frac{43}{35}=1\tfrac{8}{35}.$$

Hence, to add fractions, we have this

RULE.

Reduce the fractions to a common denominator, and take the sum of their numerators, under which place the common denominator, and it will give the sum required.

NOTE.—The labor will be the least when we reduce the fractions to their *least common denominator.*

EXAMPLES.

1. What is the sum of $\tfrac{1}{2}$, $\tfrac{1}{3}$, $\tfrac{1}{4}$, and $\tfrac{1}{6}$?

These fractions, when reduced to their least common denominator, are $\tfrac{6}{12}$, $\tfrac{4}{12}$, $\tfrac{3}{12}$, and $\tfrac{2}{12}$, the sum of whose numerators is $6+4+3+2=15$. Hence we have

$$\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{6}=\tfrac{15}{12}=\tfrac{5}{4}=1\tfrac{1}{4}.$$

2. What is the sum of $\tfrac{1}{6}$ and $\tfrac{1}{8}$? *Ans.* $\tfrac{7}{24}$.

3. What is the sum of $\tfrac{1}{5}$, $\tfrac{1}{10}$, $\tfrac{3}{20}$. *Ans.* $\tfrac{9}{20}$.

4. What is the sum of $\tfrac{1}{4}$, $\tfrac{3}{8}$, $\tfrac{7}{12}$? *Ans.* $\tfrac{29}{24}=1\tfrac{5}{24}$.

5. What is the sum of $\tfrac{4}{15}$, $\tfrac{5}{12}$, $\tfrac{3}{10}$? *Ans.* $\tfrac{59}{60}$.

6. What is the sum of $\tfrac{3}{5}$, $\tfrac{4}{15}$, $\tfrac{7}{10}$, $\tfrac{9}{30}$? *Ans.* $\tfrac{56}{30}=1\tfrac{13}{15}$.

NOTE:—If any of the fractions are compound, they must first be reduced to simple fractions, (by Rule under ART. 39.)

7. *What is the sum of* $\tfrac{1}{2}$ of $\tfrac{3}{4}$ of $\tfrac{4}{5}$, $\tfrac{1}{3}$ of $\tfrac{1}{4}$, and $\tfrac{3}{8}$?

These fractions, when reduced to their simplest forms, are $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{3}{8}$; which, when reduced to their least common denominator, become

$$\frac{4}{24},\ \frac{2}{24},\ \frac{9}{24}.$$

Hence, their sum is

$$\frac{4+2+9}{24}=\frac{15}{24}=\frac{5}{8}.$$

8. What is the sum of $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{10}{12}$, $\frac{2}{3}$ of $\frac{3}{4}$ of 6, and $\frac{1}{2}$ of $\frac{6}{7}$ of 3? *Ans.* $4\frac{13}{21}$.

9. What is the sum of $\frac{3}{5}$ of $\frac{5}{4}$ of 8, $\frac{1}{5}$ of $\frac{4}{5}$ of $\frac{10}{8}$, and $\frac{1}{8}$ of 16? *Ans.* $8\frac{1}{5}$.

10. What is the sum of $\frac{1}{4}$ of $\frac{5}{8}$ of $\frac{2}{5}$, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{1}{5}$ of $\frac{5}{8}$? *Ans.* $\frac{7}{16}$.

11. What is the sum of $\frac{3}{8}$, $\frac{4}{7}$, $\frac{5}{9}$, and $\frac{3}{4}$?

Ans. $\frac{1135}{504}=2\frac{127}{504}$.

12. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{11}{12}$?

Ans. $\frac{44}{12}=3\frac{2}{3}$.

13. What is the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, and $\frac{19}{20}$?

Ans. $\frac{78}{20}=3\frac{9}{10}$.

14. What is the sum of $3\frac{1}{2}$, $\frac{1}{4}$ of $\frac{1}{5}$, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, and $\frac{1}{15}$?

Ans. $\frac{241}{60}=4\frac{1}{60}$.

15. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{6}{7}$, and $\frac{8}{9}$ of $\frac{9}{10}$? *Ans.* $\frac{257}{105}=2\frac{47}{105}$.

16. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$?

Ans. $\frac{4609}{2520}=1\frac{2089}{2520}$.

17. What is the sum of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$?

Ans. $\frac{137}{60}=2\frac{17}{60}$.

18. What is the sum of $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$? *Ans.* $\frac{317}{60}=5\frac{17}{60}$.

19. What is the sum of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$?

Ans. $\frac{485333}{360360}=1\frac{124973}{360360}$.

20. *What is* the sum of $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{11}{10}$, $\frac{12}{11}$, $\frac{13}{12}$, $\frac{14}{13}$?

Ans. $\frac{4088933}{360360}=11\frac{124973}{360360}$

SUBTRACTION OF FRACTIONS.

44. SUPPOSE we wish to subtract $\frac{1}{7}$ from $\frac{1}{5}$. We know that so long as these fractions have different denominators, the one cannot be subtracted from the other any more than pounds can be subtracted from yards. We therefore reduce them to a common denominator, and obtain $\frac{1}{5}=\frac{7}{35}$; $\frac{1}{7}=\frac{5}{35}$. Now, taking their difference, we obtain $\frac{1}{5}-\frac{1}{7}=\frac{7}{35}-\frac{5}{35}=\frac{2}{35}$. Hence, to subtract one fraction from another we have this

RULE.

Reduce the fractions to a common denominator; subtract the less numerator from the greater, and place the common denominator under the difference.

Repeat this Rule.

EXAMPLES.

1. From $\frac{1}{2}$ subtract $\frac{1}{5}$. $\frac{1}{2}=\frac{5}{10}$; $\frac{1}{5}=\frac{2}{10}$; $5-2=3$. Therefore we have $\frac{1}{2}-\frac{1}{5}=\frac{3}{10}$.

2. From $\frac{1}{2}$ subtract $\frac{1}{3}$. *Ans.* $\frac{1}{6}$.
3. From $\frac{1}{2}$ subtract $\frac{1}{8}$. *Ans.* $\frac{3}{8}$.
4. From $\frac{3}{7}$ subtract $\frac{1}{4}$. *Ans.* $\frac{5}{28}$.
5. From $\frac{11}{12}$ subtract $\frac{1}{6}$. *Ans.* $\frac{3}{4}$.
6. From $\frac{1}{18}$ subtract $\frac{1}{360}$. *Ans.* $\frac{19}{360}$.
7. From $\frac{109}{105}$ subtract $\frac{17}{35}$. *Ans.* $\frac{58}{105}$.
8. From $\frac{360}{480}$ subtract $\frac{51}{120}$. *Ans.* $\frac{13}{40}$.

NOTE.—As in Addition, if either of the fractions is compound, it must first be reduced to its simplest form.

9. From $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{5}$ subtract $\frac{1}{10}$. *Ans.* $\frac{1}{2}$.
10. From $\frac{6}{7}$ subtract $\frac{5}{9}$ of $\frac{18}{25}$. *Ans.* $\frac{16}{35}$.
11. From $\frac{1}{2}$ of $\frac{3}{7}$ subtract $\frac{1}{7}$ of $\frac{3}{8}$. *Ans.* $\frac{9}{56}$.
12. *From* $3\frac{1}{5}$ *subtract* $2\frac{1}{9}$. Ans. $1\frac{4}{45}$.

13. From $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$, subtract $\frac{1}{8}$ of $\frac{2}{7}$. *Ans.* $\frac{37}{140}$.

14. From $\frac{3}{2}$ of $\frac{4}{3}$ of $\frac{5}{4}$ of $\frac{6}{5}$, subtract $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$. *Ans.* $2\frac{2}{3}$.

15. From the sum of $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, subtract the sum of $\frac{1}{2}$, $\frac{1}{3}$, , $\frac{1}{5}$. *Ans.* 4.

16. From the sum of $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{11}{10}$, $\frac{12}{11}$, $\frac{13}{12}$, $\frac{14}{13}$, ubtract the sum of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$. *Ans.* 10.

17. From $\frac{3}{2}$ of 137 subtract $\frac{1}{2}$ of 317. *Ans.* 47.

18. From $\frac{4}{3}$ of 137 subtract $\frac{1}{3}$ of 317. *Ans.* 77.

19. From $\frac{5}{4}$ of 137 subtract $\frac{1}{4}$ of 317. *Ans.* 92.

20. From $\frac{6}{5}$ of 137 subtract $\frac{1}{5}$ of 317 *Ans.* 101.

MULTIPLICATION OF FRACTIONS.

45. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

We know, (Art **39**,) that $\frac{2}{3}$ multiplied by $\frac{4}{5}$ is the ame as $\frac{2}{3}$ of $\frac{4}{5}$. Hence, we must use the same rule as for ducing compound fractions.

Therefore, to multiply fractions, we have this

RULE.

Multiply all the numerators together for a new numerator, ıd all the denominators together for a new denominator; ways observing to reject or cancel such factors as are mmon to both numerators and denominators.

If any of the factors are whole numbers, they may be *de to take* the form of a fraction by giving to them 1 for a denominator, (see ART. **38**,) and then the general rule will apply.

What is the Rule for multiplying fractions?

EXAMPLES.

1. Multiply $\frac{1}{2}$ by $\frac{1}{3}$. *Ans.* $\frac{1}{6}$.
2. Multiply $\frac{1}{2}$ by $\frac{1}{4}$. *Ans.* $\frac{1}{8}$.
3. Multiply $\frac{1}{3}$ by $\frac{2}{5}$. *Ans.* $\frac{2}{15}$.
4. Multiply $\frac{1}{3}$ by $\frac{7}{9}$. *Ans.* $\frac{7}{27}$.
5. Multiply $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, all together. *Ans.* $\frac{1}{40}$.
6. Multiply $\frac{4}{5}$ by $\frac{10}{16}$.

In this example, we cancel the 4 of the numerator against a corresponding factor of the 16 of the denominator; and 5 of the denominator against a corresponding factor of the 10 in the numerator. Thus:

$$\frac{\cancel{4}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{\underset{4}{\cancel{16}}}$$

Finally, cancelling the 2 in the numerator against the same factor of the 4 in the denominator, we find

$$\frac{\cancel{4}}{\cancel{5}} \times \frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}} = \frac{1}{2}\ \textit{Ans.}$$

7. Multiply the fractions $\frac{3}{4}$, $\frac{8}{9}$, $\frac{6}{7}$.

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\underset{\cancel{3}}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{7} = \frac{4}{7}\ \textit{Ans.}$$

NOTE.—A little practice will enable the student to perform these operations of cancelling with great ease and rapidity. And since, as was remarked under ART. **39**, it is immaterial which factors are first cancelled, the simplicity of the work must depend much upon his skill and ingenuity.

8. Multiply together the fractions $3\frac{1}{2}$, $4\frac{1}{3}$, $\frac{1}{14}$.

Expressing the multiplication, after reducing them, we have

$$\frac{7}{2}\times\frac{13}{3}\times\frac{1}{14}.$$

Cancelling the 7 of the numerator against a part of the 14 of the denominator, we have

$$\frac{\cancel{7}}{2}\times\frac{13}{3}\times\frac{1}{\underset{2}{\cancel{14}}}=\frac{13}{12}=1\frac{1}{12}.\ \textit{Ans.}$$

9. Multiply together the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. *Ans.* $\frac{1}{5}$.

10. Multiply together the fractions $\frac{3}{7}$, $\frac{4}{5}$, $\frac{5}{9}$, $\frac{14}{3}$. *Ans.* $\frac{8}{9}$.

11. Multiply together the fractions $3\frac{1}{3}$, $4\frac{1}{4}$, $5\frac{1}{5}$.
Ans. $\frac{221}{3}=73\frac{2}{3}$.

12. Multiply together $\frac{3}{40}$, $\frac{7}{18}$, $\frac{5}{7}$. *Ans.* $\frac{1}{48}$.

13. Multiply $\frac{3}{7}$ by 4. *Ans.* $\frac{12}{7}=1\frac{5}{7}$.

14. Multiply 7 by $\frac{3}{4}$. *Ans.* $\frac{21}{4}=5\frac{1}{4}$.

15. Multiply $7\frac{1}{2}$ by $3\frac{1}{2}$. *Ans.* $\frac{105}{4}=26\frac{1}{4}$.

16. Multiply $16\frac{1}{2}$ by 5, *Ans.* $\frac{165}{2}=82\frac{1}{2}$.

17. Multiply the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, by the sum of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$. *Ans.* $\frac{1463}{1200}=1\frac{263}{1200}$.

18. Multiply the sum of $\frac{1}{2}$ of $\frac{2}{4}$, $\frac{3}{4}$ of $\frac{3}{5}$ by the sum of $\frac{1}{3}$, of $\frac{1}{4}$, $\frac{1}{5}$ of $\frac{1}{6}$. *Ans.* $\frac{49}{600}$.

19. Multiply $\frac{3}{7}$ of $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{4}{11}$ by $\frac{11}{12}$ of $\frac{6}{7}$ of $\frac{8}{9}$.
Ans. $\frac{5}{63}$.

20. *Multiply* the sum of 3, $3\frac{1}{2}$, $3\frac{1}{3}$, $3\frac{1}{4}$, by the sum of $2\frac{1}{3}$ $3\frac{1}{4}$, $4\frac{1}{5}$. Ans. $\frac{99203}{720}=137\frac{563}{720}$

DIVISION OF FRACTIONS.

46. Let us endeavor to divide $\frac{4}{7}$ by $\frac{5}{8}$. We know that $\frac{4}{7}$ can be divided by 5, by multiplying the denominator by 5, (see PROP. II., ART. **34**,) which gives

$$\frac{4}{7\times5}.$$

Now, since $\frac{5}{8}$ is but one eighth of 5, it follows that $\frac{4}{7}$ divided by $\frac{5}{8}$ must give a quotient eight times as great as $\frac{4}{7}$ divided by 5. Therefore, $\frac{4}{7}$ divided by $\frac{5}{8}$ must give

$$8 \text{ times } \frac{4}{7\times5}=\frac{4\times8}{7\times5}.$$

From which we see that $\frac{4}{7}$ has been multiplied by $\frac{8}{5}$ after it was inverted.

Hence, to divide one fraction by another, we have this

RULE.

Reduce the fractions to their simplest form. Invert the divisor, and then proceed as in multiplication.

If either the dividend or divisor is a whole number, it may be converted into an improper fraction having 1 for its denominator.

Repeat the Rule for the Division of Fractions.

EXAMPLES.

1. Divide $\frac{12}{13}$ by $\frac{4}{26}$.

Inverting the divisor, we have

$$\frac{12}{13}\times\frac{26}{4}.$$

Cancelling, we find

$$\frac{\overset{3}{\cancel{12}}}{\cancel{13}}\times\frac{\overset{2}{\cancel{26}}}{\cancel{4}}=6.$$

2. Divide $\frac{1}{2}$ by $\frac{1}{3}$. *Ans.* $\frac{3}{2}=1\frac{1}{2}$.
3. Divide $\frac{1}{2}$ by $\frac{1}{4}$. *Ans.* 2.
4. Divide $\frac{3}{4}$ by $\frac{1}{3}$. *Ans.* $\frac{9}{4}=2\frac{1}{4}$.
5. Divide $\frac{6}{7}$ by $\frac{6}{11}$. *Ans.* $\frac{11}{7}=1\frac{4}{7}$.
6. Divide $\frac{8}{15}$ by $\frac{12}{20}$. *Ans.* $\frac{8}{9}$.
7. What is the quotient of $4\frac{1}{3}$ divided by $17\frac{1}{2}$? *Ans.* $\frac{26}{105}$.
8. What is the quotient of $\frac{15}{17}$ divided by 10? *Ans.* $\frac{3}{34}$.
9. Divide $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{1}{6}$ of $\frac{5}{8}$. *Ans.* $\frac{18}{5}=3\frac{3}{5}$.
10. Divide $3\frac{1}{3}$ of $2\frac{1}{2}$ by $4\frac{1}{4}$. *Ans.* $\frac{100}{51}=1\frac{49}{51}$.
11. Divide $\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{4}{5}$. *Ans.* $\frac{5}{3}=1\frac{2}{3}$.
12. Divide the sum of $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, by the sum of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. *Ans.* $\frac{317}{137}$.
13. Divide the sum of $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{11}{10}$, $\frac{12}{11}$, $\frac{13}{12}$, $\frac{14}{13}$, by the sum of 1, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$. *Ans.* $\frac{4089033}{845793}=4\frac{705861}{845793}$.
14. Divide $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{1}{7}$ of $\frac{2}{3}$. *Ans.* $\frac{18}{5}=3\frac{3}{5}$.
15. Divide the sum of 1, $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, by the sum of $1\frac{1}{3}$, $2\frac{1}{4}$, $3\frac{1}{5}$. *Ans.* $\frac{485}{407}=1\frac{78}{407}$.
16. Divide the sum of $\frac{1}{3}$ of $\frac{4}{5}$, $\frac{1}{4}$ of $\frac{5}{6}$, by the sum of $\frac{1}{5}$ of $\frac{6}{7}$, $\frac{1}{6}$ of $\frac{7}{8}$. *Ans.* $\frac{798}{533}=1\frac{265}{533}$.
17. Divide $\frac{2}{5}$ of $\frac{16}{11}$ of $\frac{22}{27}$ by $\frac{1}{6}$ of $\frac{3}{4}$ of $\frac{4}{9}$. *Ans.* $\frac{128}{15}8\frac{8}{15}$.
18. Divide $\frac{7}{8}$ of $\frac{16}{27}$ of $\frac{1}{2}$ by $\frac{5}{8}$ of $\frac{2}{3}$ of 12. *Ans.* $\frac{7}{135}$.
19. Divide $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ by $\frac{1}{3}$ of $\frac{1}{5}$ of 8. *Ans.* $\frac{525}{1024}$.

RECIPROCALS OF NUMBERS.

47. The *reciprocal* of a number is the result obtained by dividing 1 by the number. Thus, the reciprocals of 2, 3, 4, and 5, are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. From this we discover that the reciprocal of an *integer*, or whole number, is equal to a vulgar fraction whose numerator is 1, and whose denominator is the given number.

The reciprocal of $\frac{3}{5}$ is found by dividing 1 by $\frac{3}{5}$, which (ART. **46**,) is $1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$.

In the same way we find the reciprocal of $\frac{7}{8}$ to be $\frac{8}{7}$, and in general, the reciprocal of a vulgar fraction is the value of the fraction when *inverted*.

NOTE.—From this, we see that dividing by any number is in effect the same as multiplying by the reciprocal of that number. So that operations of division may be included under those of multiplication. A practical application of this principle may be seen under Reduction of Denominate Fractions. (ART. **89.**)

EXAMPLES.

1. What are the reciprocals of 7, 8, 9, 10, 11?
Ans. $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.
2. What are the reciprocals of 18, 23, and 41?
Ans. $\frac{1}{18}$, $\frac{1}{23}$, $\frac{1}{41}$.
3. What are the reciprocals of $\frac{3}{8}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$? *Ans.* $\frac{8}{3}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$.
4. What are the reciprocals of $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$? *Ans.* $\frac{2}{3}$, $\frac{3}{7}$, $\frac{4}{13}$.
5. What are the reciprocals of $\frac{3}{4}$ of $\frac{8}{9}$, $\frac{2}{3}$ of $\frac{7}{8}$?
Ans. $\frac{4}{3}$ of $\frac{9}{8}$, $\frac{3}{2}$ of $\frac{8}{7}$.

48. EXERCISES IN VULGAR FRACTIONS.

1. Reduce $\frac{24}{48}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
2. Reduce $\frac{8}{64}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
3. Reduce $\frac{14}{16}$ to its lowest terms. *Ans.* $\frac{7}{8}$.

4. Reduce $\frac{588}{672}$ to its lowest terms. *Ans.* $\frac{7}{8}$.

5. Reduce $\frac{3387}{4516}$ to its lowest terms. *Ans.* $\frac{3}{4}$.

6. Reduce $\frac{7721}{7763}$ to its lowest terms. *Ans.* $\frac{1103}{1109}$.

7. Reduce $\frac{15636}{16939}$ to its lowest terms *Ans.* $\frac{12}{13}$.

8. Reduce $\frac{505}{515}$ to its lowest terms. *Ans.* $\frac{101}{103}$.

9. Reduce $\frac{18999}{21110}$ to its lowest terms. *Ans.* $\frac{9}{10}$.

10. Reduce $\frac{515}{505}$ to a mixed number. *Ans.* $1\frac{2}{101}$.

11. Reduce $\frac{37}{5}$ to a mixed number. *Ans.* $7\frac{2}{5}$.

12. Reduce $\frac{48}{6}$ to a whole number. *Ans.* 8.

13. Reduce $\frac{59}{17}$ to a mixed number. *Ans.* $3\frac{8}{17}$.

14. Reduce $\frac{2371}{907}$ to a mixed number. *Ans.* $2\frac{557}{907}$.

15. Reduce $\frac{101}{97}$ to a mixed number. *Ans.* $1\frac{4}{97}$.

16. Reduce $3\frac{1}{2}$ to an improper fraction. *Ans.* $\frac{7}{2}$.

17. Reduce $15\frac{11}{13}$ to an improper fraction. *Ans.* $\frac{206}{13}$.

18. Reduce $3\frac{7}{17}$ to an improper fraction. *Ans.* $\frac{58}{17}$.

19. Reduce $1\frac{8}{101}$ to an improper fraction. *Ans.* $\frac{109}{101}$.

20. Reduce $100\frac{11}{17}$ to an improper fraction. *Ans.* $\frac{1711}{17}$.

21. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to its simplest form. *Ans.* $\frac{1}{4}$.

22. Reduce $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{20}{27}$ to its simplest form. *Ans.* $\frac{1}{3}$.

23. Reduce $\frac{7}{8}$ of $\frac{1}{2}$ of $\frac{6}{14}$ of 3 to its simplest form. *Ans.* $\frac{9}{16}$.

24. Reduce $\frac{1}{10}$ of $\frac{3}{7}$ of $\frac{21}{25}$ of $3\frac{1}{3}$ to its simplest form.
Ans. $\frac{3}{25}$.

25. Reduce $\frac{6}{7}$ of $\frac{14}{3}$ of $\frac{1}{2}$ of 100 to its simplest form.
Ans. 200.

26. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, to equivalent fractions having a common denominator. *Ans.* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$.

27. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, to equivalent fractions having a common denominator. *Ans.* $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{10}{60}$.

28. Reduce $3\frac{1}{2}$, $\frac{7}{5}$, $\frac{3}{4}$, $\frac{3}{10}$, to equivalent fractions having a common denominator. *Ans.* $\frac{70}{20}$, $\frac{28}{20}$, $\frac{15}{20}$, $\frac{6}{20}$.

29. Reduce $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{11}$, to equivalent fractions having a *common denominator.* *Ans.* $\frac{385}{1155}$, $\frac{231}{1155}$, $\frac{165}{1155}$, $\frac{105}{1155}$

30. Reduce $\frac{3}{5}$, $\frac{5}{7}$, $\frac{7}{11}$, $\frac{11}{13}$, to equivalent fractions having a common denominator. *Ans.* $\frac{3003}{5005}$, $\frac{3575}{5005}$, $\frac{3185}{5005}$, $\frac{4235}{5005}$.

31. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$? *Ans.* $\frac{13}{12}=1\frac{1}{12}$.

32. What is the sum of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$? *Ans.* $\frac{143}{60}=2\frac{23}{60}$.

33. From a piece of cloth $\frac{1}{2}$ and $\frac{1}{3}$ of the whole was cut off. What part of the whole was thus taken away?

Ans. $\frac{5}{6}$.

34. From $\frac{1}{2}$ subtract $\frac{1}{3}$. *Ans.* $\frac{1}{6}$.

35. From $\frac{1}{10}$ subtract $\frac{1}{11}$. *Ans.* $\frac{1}{110}$.

36. From $\frac{3}{4}$ subtract $\frac{4}{7}$. *Ans,* $\frac{5}{28}$.

37. A tree 150 feet high had $\frac{1}{5}$ broken off in a storm. What was the length broken off? *Ans.* 30 feet.

38. A and B together possess 1477 sheep, of which A owns $\frac{4}{7}$ and B $\frac{3}{7}$. How many belong to each man?

Ans. { A's, 844.
B's, 633.

39. A owns $\frac{3}{11}$ of a ship, valued at $15422; he sells to B $\frac{2}{3}$ of his share. What is the value of what A has left; also, what is the value of B's part?

Ans. { A's remaining part is $1402.
B's part is $2804.

40. A cotton mill is sold for $30000, of which A owns $\frac{1}{5}$ of the whole, B and C each own $\frac{1}{2}$ of $\frac{1}{3}$ of the whole. How many dollars does each one claim?

Ans. { A claims $6000.
B claims $5000.
C claims $5000.

41. A and B have a melon, of which A owns $\frac{3}{8}$, and B $\frac{5}{8}$; C offers them one shilling, to partake equally with them of the melon, which was agreed to. How must the shilling be divided between A and B?

Ans. { A must have $\frac{1}{8}$ of it.
B must have $\frac{7}{8}$ of it.

42. A farmer had $\frac{1}{5}$ of his sheep in one field, $\frac{1}{4}$ in a second field, and the residue, which was 779, in a third field. How many sheep had he in all? *Ans.* 1230 sheep.

43. If I divide 616 dollars between A, B, C, and D, by giving A $\frac{1}{4}$ of the whole, B $\frac{5}{14}$ of the remainder, C $\frac{8}{9}$ of what then remained, and D the balance, how much will each receive?

Ans. {
A had 154 dollars.
B " 165 "
C " 264 "
D " 33 "

DECIMAL FRACTIONS.

49. A Decimal Fraction is that particular form of a Fraction, whose denominator consists of a unit, followed by one or more ciphers.

Thus: $\frac{1}{10}$, $\frac{3}{10}$, $\frac{4}{100}$, $\frac{37}{100}$, $\frac{8}{100}$, $\frac{3}{1000}$, $\frac{47}{10000}$, &c., are Decimal Fractions.

In practice, the denominators of Decimal Fractions are not written, but always understood.

The above Decimal Fractions are usually written as follows: 0·1, 0·3, 0·04, 0·37, 0·08, 0·003, 0·0047, &c.

The period, or decimal point, serves to separate the decimals from the whole numbers.

The first figure on the right of the *decimal point*, is in the place of tenths; the second figure is in the place of hundredths; the third figure in the place of thousandths, and so on; the value of the units of the successive figures decreasing from the left towards the right, in a tenfold *ratio, as in* whole numbers. The following table will *exhibit this.*

NUMERATION TABLE OF WHOLE NUMBERS AND DECIMALS.

&c. &c.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.	Ten Billionths.	&c., &c
	3	3	3	3	3	3	3	3	3	3	3	.	3	3	3	3	3	3	3	3	3	3	

Ascending. ☜ ☞ Descending.

This table is in accordance with the French method of numeration (ART. 6,) where each period of three figures changes its name and value.

Since decimals, like whole numbers, decrease from the left towards the right in a ten-fold ratio, they may be connected together by means of the decimal point, and then operated upon by precisely the same rules as for whole numbers, provided we are careful to keep the decimal point always in the right place.

Annexing a cipher to a decimal does not change its value, because it is the same as multiplying its numerator and denominator by 10. Thus: 0·3=0·30=0·300=&c. But prefixing a cipher is the same as removing the decimal figures one place farther to the right, and therefore each cipher, thus prefixed, reduces the value in a ten-fold ratio.

Thus: 0·3 is ten times 0·03, or a hundred times 0·003.

0·2 is read two tenths.
0·25 " twenty-five hundredths.
0·365 " three hundred and sixty-five thousandths.
0·105 " one hundred and five thousandths.
0·03 " three hundredths.

0·1234 is read one thousand two hundred and thirty-four ten thousandths.
4·3 " four and three tenths.
37·3 " thirty-seven and three tenths.
365·03 " three hundred and sixty-five and three hundredths.
&c. &c.

6·4 is the same as $6\frac{4}{10}=\frac{64}{10}$.
36·5 " $36\frac{5}{10}=\frac{365}{10}$.
36·05 " $36\frac{5}{100}=\frac{3605}{100}$.
0·7 " $\frac{7}{10}$.
0·37 " $\frac{37}{100}$.
0·123 " $\frac{123}{1000}$
0·2345 " $\frac{2345}{10000}$.
0·0101 " $\frac{101}{10000}$.
0·00012 " $\frac{12}{100000}$.
0·40056 " $\frac{40056}{100000}$.
40·0005 " $40\frac{5}{10000}=\frac{400005}{10000}$.
101·0101 " $101\frac{101}{10000}=\frac{1010101}{10000}$.

A number composed of a whole number and a decimal part, is called a *mixed number*.

What is a decimal fraction? Of what form is the denominator? Give examples of decimal fractions. In practice, which part is not written, but understood? What purpose does the decimal point serve? What place is the first figure on the right of the decimal point said to occupy? What place does the second figure occupy? What place does the third figure occupy? In what ratio do the values decrease in passing to the right? Is the above table in accordance with the French or English method of notation? Does annexing a cipher to a decimal alter its value? What effect is produced by prefixing a cipher? A number which is composed of a whole number and decimal is called, what?

Let the pupil be exercised in decompounding decimals, as we have done in the following

EXAMPLES.

The expression, $\frac{7468}{1000}$, implies that 7468 is to be divided by 1000. Performing the division by the method of CASE IV, ART. **30**, we obtain 7 for a quotient and 468 for a remainder. So that $\frac{7468}{1000}=7\frac{468}{1000}=7\cdot468=7$ units, 4 tenths, 6 hundredths, 8 thousandths, or, which is the same thing, it equals $7+\frac{4}{10}+\frac{6}{100}+\frac{8}{1000}$.

In a similar manner we find that

$$\frac{364587}{10}=36458\cdot7=36458+\frac{7}{10}.$$

$$\frac{364587}{100}=3645\cdot87=3645+\frac{8}{10}+\frac{7}{100}.$$

$$\frac{364587}{1000}=364\cdot587=364+\frac{5}{10}+\frac{8}{100}+\frac{7}{1000}.$$

$$\frac{364587}{10000}=36\cdot4587=36+\frac{4}{10}+\frac{5}{100}+\frac{8}{1000}+\frac{7}{10000}.$$

$$\frac{364587}{100000}=3\cdot64587=3+\frac{6}{10}+\frac{4}{100}+\frac{5}{1000}+\frac{8}{10000}+\frac{7}{100000}.$$

$$\frac{364587}{1000000}=0\cdot364587=\frac{3}{10}+\frac{6}{100}+\frac{4}{1000}+\frac{5}{10000}+\frac{8}{100000}+\frac{7}{1000000}.$$

ADDITION OF DECIMAL FRACTIONS.

50. SINCE decimals, like whole numbers, increase from the right towards the left, they may be treated by the same rules as for whole numbers, provided we are careful to keep the decimal point in the right place, so that like orders may stand under each other. Hence we have this

RULE.

Place the numbers so that the decimal points shall be directly under each other; add as in whole numbers. In the amount place the point under the points in the numbers added.

How do you place the numbers to be added? How, the point in the amount?

EXAMPLES.

(1.)

	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.
	3	7	4	1 ·	1	2	5	0
	1	4	1	2 ·	1	3	4	6
	2	3	1	0 ·	2	0	0	5
	4	1	0	0 ·	1	0	1	6
	3	4	5	6 ·	4	3	1	2
1	5	0	1	9 ·	9	9	2	9

(2.)

	Units.	Tenths.	Hundredths.	Thousandths.	Tens of Thousandths.	Hund. Thousandths.
	1 ·	2	3	4	5	6
	2 ·	3	4	5	6	7
	3 ·	4	5	6	7	8
	4 ·	5	6	7	8	9
	5 ·	6	7	8	0	0
1	7 ·	2	8	2	9	0

(3.)	(4.)	(5.)
4123·245	0·43478	11·111
1·12	1·35001	210·001
37·004	1·1	8·8
0·205	33·333	9·808
4161·574	36·21779	239·720

6. What is the sum of 0·123, 0·012, 0·675, 0·0045?
Ans. 0·8145.

7. What is the sum of 0·14145, 0·23235, 0·343 0·45455? *Ans.* 1·1718.

8. Find the sum of 1·0012, 23·1003, 101·314 10·101578. *Ans.* 135·517148

9. Find the sum of 234·12, 23·412, 2·3412, 0·23412
Ans. 260·10732.

10. What is the sum of 111·111, 12·1212, 13·1313, 14·1414? *Ans.* 150·5049

SUBTRACTION OF DECIMAL FRACTIONS.

51. There is no difference between the subtraction of decimals and that of whole numbers, provided we are careful to keep the decimal points directly under each other, so that like orders may stand under each other. Hence this

RULE.

Place the less number under the greater, so that the decimal points shall be directly under each other; subtract, as in whole numbers. In the difference place the point under the points of the numbers above.

How do you place the numbers in subtraction? Then how do you proceed?

EXAMPLES.

(1.)	(2.)	(3.)
345·345	1245·3478	3456·12347846
54·123	340·0122	479·100345
291·222	905·3356	2977·02313346

4. From 1023·4 subtract 99·9. *Ans.* 923·5.
5. *From 0·4785* subtract 0·13047. Ans. 0·34803
6 *From 0·11234*,subtract 0·00675. Ans. 0·105

MULTIPLICATION OF DECIMAL FRACTIONS.

52. A tenth taken once, must give 1 *tenth* for a product; if taken only one-tenth of a time, the product will be one-tenth of a tenth, or one hundredth; that is, $\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$, or decimally expressed $0{\cdot}1\times0{\cdot}1=0{\cdot}01$. This is evidently true, since if the tenth-part of any thing be divided into 10 equal parts, each subdivision will be a hundredth-part of the whole. So $\frac{1}{10}$ of $\frac{1}{100}=\frac{1}{1000}$, and so on.

Multiply 0·136 by 0·78. If we supply the denominators of these decimal fractions, which denominators are always understood, we shall have

$$0{\cdot}136=\frac{136}{1000};\ 0{\cdot}78=\frac{78}{100}.$$

Hence, multiplying $\frac{136}{1000}$ by $\frac{78}{100}$, (ART. **45**,) we find

$$\frac{136}{1000}\times\frac{78}{100}=\frac{136\times78}{1000000}=\frac{10608}{1000000}=0{\cdot}10608.$$

From which we see that the number of decimal places in the product, always denoted by the number of zeros in the denominator, which is understood, is equal to the number of decimal places in both factors. Hence we have this

RULE.

Multiply as in whole numbers, and give as many decimal places in the product as there are in both the factors. When there are not as many places in the product, prefix ciphers.

How do you multiply decimals? How many decimal places must there be in the product? When the whole number of figures in the product is not as great, how do you proceed?

EXAMPLES.

1. Multiply 0·125 by 0·37.

OPERATION.

$$\begin{array}{r} 0{\cdot}125 \\ 0{\cdot}37 \\ \hline 875 \\ 375 \\ \hline 0{\cdot}04625 \end{array}$$

In this example, the multiplicand has 3 decimal places and the multiplier has 2; therefore, by the rule, the produc must have 5 places, and since the product consists of but figures, we prefix one cipher before making the decimal point

2. Multiply 0·561 by 0·786. *Ans.* 0·440946.
3. Multiply 3·012 by 4·027. *Ans.* 12·129324.
4. Multiply 47·051 by 37·039. *Ans.* 1742·721989.
5. Multiply 33·33 by 66·66. *Ans.* 2221·7778.
6. Multiply 125·125 by 5·5. *Ans.* 688·1875.

53. A decimal number may be multiplied by 10, 10 1000, &c., by removing the decimal point as many place to the right as there are ciphers in the multiplier; and i there are not so many figures, make up the deficiency b annexing ciphers.

$$\text{Thus, } 12{\cdot}12 \text{ multiplied by} \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \\ 1000000 \end{array}\right\} = \left\{\begin{array}{r} 121{\cdot}2 \\ 1212. \\ 12120. \\ 121200. \\ 1212000. \\ 12120000. \end{array}\right.$$

How may a decimal number be multiplied by 10, 100, 1000, &c.? When there a not as many decimal figures in the multiplicand as there are ciphers in the multipli how do you proceed?

DIVISION OF DECIMAL FRACTIONS.

54. In multiplication of decimals, we know that th number of decimal places in the product is equal to th sum of those in both the factors. Now, since the produc *divided by one of* the factors must produce the other f tor or quotient, it follows, that in division the decimal places of the dividend must be equal to the number of places in the divisor and quotient taken together. Hence, the number of decimal places in the quotient must equal the excess of those in the dividend above those in the divisor.

Divide 5·81224 by 5·432.

Dividing 581224 by 5432 we find 107 for the quotient. Since 5 figures of the dividend are decimals, and only 3 figures of the divisor are decimals, it follows that two figures of the quotient 107 must be decimals, so that 1·07 is the quotient sought.

Hence the following

RULE.

Divide as in whole numbers; give as many decimal places in the quotient as those in the dividend exceed those in the divisor; if there are not as many, supply the deficiency by prefixing ciphers.

How do you divide one decimal by another? How many decimal places must the quotient have? If the whole number of figures in the quotient is not as great as the number of decimals required, how do you proceed?

EXAMPLES.	OPERATION.
1. Divide 0·123428 by 11·8.	11·8)0·123428(0·01046 118 542 472 708 708

In this example, the dividend contains 6 decimal places, and the divisor but 1; therefore, by the rule, the quotient *ought to contain* 5; but as there are but 4 figures in the

quotient, we make up the deficiency by prefixing a cipher before making the decimal point.

2. Divide 3·810688 by 1·12. *Ans*. 3·4024.
3. Divide 0·109896 by 0·241. *Ans*. 0·456.
4. Divide 1·12264556 by 1·0012. *Ans*. 1·1213.
5. Divide 0·01764144 by 0·0018. *Ans*. 9·8008.

55. When there are not as many decimal places in the dividend as in the divisor, we may, (by ART. **49**,) annex as many ciphers to the dividend as we please, if we do not change the place of the decimal point. When the number of decimal places is the same in both dividend and divisor, the quotient will be a whole number. As for example, $\frac{6}{10}$ divided by $\frac{2}{10}=3$, which is a whole number; that is, 0·6 divided by 0·2=3, a whole number.

When there are not as many decimal places in the dividend as in the divisor, how do you proceed? When the number of decimal places in the dividend is the same as in the divisor, what will the quotient be?

6. Divide 244·431 by 1·2345.

In this example, before performing the division, we annex a cipher to the dividend, so that it may have as many decimal places as the divisor has; we then perform this

OPERATION.

```
1·2345)244·4310(198 whole
       12345        number.
       -----
       120981
       111105
       ------
        98760
        98760
        -----
```

7. Divide 122·418 by 3·4005 *Ans*. 36.
8. Divide 0·7 by 0·07. *Ans*. 10.
9. Divide 0·25 by 0·0005. *Ans*. 500.
10. Divide 0·125 by 0·000005. Ans. 25000.

56. When there is still a remainder, and we wish a more accurate quotient, we may continue to annex ciphers and to divide as far as we please, observing the rule for placing the decimal point.

11. Divide 20 by 0·003.

OPERATION.

```
By Short Division.   0·003)20·000
                           6666·6666, &c., to any extent.
```

12. Divide 37·4 by 4·5.

OPERATION.

```
4·5)37·4(8·31111+
    360
    ---
     140
     135
     ---
       50
       45
       --
        50
        45
        --
         50
         45
         --
          50
          45
          --
           5
```

When, in the quotient, we write the sign + it is to *indicate that* the quotient is still larger than is written.

It frequently happens, as in this example, that the work will never terminate.

When there is still a remainder, how may we proceed to obtain a still more accurate value for the quotient? What does the sign + at the right of a quotient indicate?

13 Divide 7·85 by 3·43. *Ans.* 2·2886+.
14 Divide 0·478 by 0·58. *Ans.* 0·824+.
15. Divide 0·9009 by 0·4051. *Ans.* 2·223+.

57. We may, obviously, divide any decimal by 10, 100, 1000, &c., by removing the decimal point as many places to the left as there are ciphers in the divisor; when there are not so many figures at the left of the decimal point, we may prefix ciphers.

$$\text{Thus, } 12{\cdot}12 \text{ divided by } \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\\1000000\end{array}\right\} = \left\{\begin{array}{l}1{\cdot}212.\\0{\cdot}1212.\\0{\cdot}01212.\\0{\cdot}001212.\\0{\cdot}0001212.\\0{\cdot}00001212.\end{array}\right.$$

How may we divide a decimal by 10, 100, 1000, &c.? When in the decimal number there are not as many figures on the left of the decimal point as there are ciphers in the divisor, how do you proceed?

FEDERAL MONEY.

58. This is the currency of the United States.

Its denominations, or names, are Eagles, Dollars, Dimes, Cents, and Mills.

$$\left.\begin{array}{l}\text{Eagles,}\\\text{Half Eagles,}\\\textit{Quarter Eagles,}\end{array}\right\}\text{ are coined from gold.*}$$

---

* See note at end of the subject of Federal Money.

Dollars,
Half Dollars,
Quarter Dollars, } are coined from silver.
Dimes,
Half Dimes,

Cents,
Half Cents, } are coined from copper.

The Mill is never coined.

59. The gold for coinage is not pure, but consists of $\frac{22}{24}$ of pure gold, $\frac{1}{24}$ of silver, and $\frac{1}{24}$ of copper; or, as usually expressed, 22 carats of gold, 1 of silver, and 1 of copper. A *carat* being $\frac{1}{24}$ part of the whole.

The standard for silver is 1489 of pure silver to 179 of pure copper; which, in carats, is $21\frac{59}{139}$ of silver, and $2\frac{80}{139}$ of copper.

The copper coins are of pure copper.

By an Act of Congress, approved January 18, 1837, the gold and silver coin was to consist of $\frac{900}{1000}=\frac{9}{10}$ pure metal, and $\frac{100}{1000}=\frac{1}{10}$ alloy. The alloy for silver was to consist of pure copper, and the alloy for gold was to consist of copper and silver, provided that the silver does not exceed one half of the whole alloy.

The weight of the Eagle was fixed at 258 grains; the weight of the Dollar was to be 412½ grains; that of the Cent was to be 168 grains.

TABLE OF FEDERAL MONEY.

10	mills *m*	make	1 cent,	*ct.*
10	cents	"	1 dime,	*d.*
10	dimes	"	1 dollar,	$.*
10	dollars	"	1 eagle,	*E.*

---

* *The symbol $ is probably a combination of the letters U. S., written $, to express United States money.*

m. *ct.*
10= 1 *d.*
100= 10= 1 $.
1000= 100= 10= 1 *E.*
10000=1000=100=10=1.

Where is Federal Money used? What are its denominations? Which are coined from gold? Which from silver? Which from copper? Which one is never coined? What metals are mixed with gold for coining? In gold coins, what is the ratio of the copper and silver to the gold? What is a carat? What is the standard for silver coins? What is the ratio when estimated in carats? Is the copper for copper coins also alloyed? By Act of Congress, 1837, what fractional part of gold and silver coin is pure metal? And what part is alloy? Of what metal is the alloy for silver? Of what metals is the alloy for gold? What is the weight of the Eagle? What is the weight of the Dollar? What the weight of the Cent? Repeat the table of Federal Money.

60. Since the different denominations succeed each other in a ten-fold ratio, as in whole numbers and decimals, it is plain that the preceding rules for decimals are applicable to this currency. The United States was the first, and only government, that adopted the decimal division for its currency. It is much to be regretted that they did not, at the same time, give the decimal division to their weights and measures. Notwithstanding the great simplicity of the decimal division, a large number of our merchants mark their goods in shillings and pence, and some even keep their book accounts in English currency. The rates of postage, having been fixed in Federal currency, will do much towards bringing about its universal use in the United States. Federal money ought never to be treated as consisting of different denominations, since it is by far the simplest and best way to consider its denominations the same as decimals. To make this more clear, we will give the following table of Federal Money:

TABLE.

&c. &c.	Thousands of dollars.	Hundreds of dollars.	Tens of dollars, or eagles.	Units or ones, or dollars.		Tenths of a dollar, or dimes.	Hundredths of a dollar, or cents.	Thousandths of a dollar, or mills.
	4	4	4	4	.	4	4	4=$4444, 44 cents, 4 mills.
		4	4	4	.	4	4	4=$444, 44 cents, 4 mills.
			4	4	.	4	4	4=$44, 44 cents, 4 mills.
				4	.	4	4	4=$4, 44 cents, 4 mills.
				0	.	4	4	4=44 cents, 4 mills.
				0	.	0	4	4=4 cents, 4 mills.
				0	.	0	0	4=4 mills.

It is customary, in accounts, to use only dollars, cents, and mills, so that eagles are expressed in dollars, and dimes in cents.

In what ratio do the different denominations of Federal Money decrease? Are the rules for decimals applicable to this currency? Should Federal Money be treated as denominate numbers? In accounts, which denominations only are used? How, then, are Eagles expressed? How are Dimes expressed?

Thus: 5 eagles and 6 dollars is the same as 56 dollars.
4 dimes and 5 cents is the same as 45 cents.
3 dimes 3 cents and 3 mills is the same as 333 mills.
2 dimes and 2 mills is the same as 202 mills.

1 dollar is the same as 100 cents, which is 1000 mills.
2 dollars is the same as 200 cents, which is 2000 mills.
5 dollars is the same as 500 cents, which is 5000 mills.
7 dollars is the same as 700 cents, which is 7000 mills.
56 dollars is the same as 5600 cents, which is 56000 mills.
365 dollars is the same as 36500 cents, which is 365000 mills.
3456 dollars is the same as 345600 cents, which is 3456000 mills.
&c. &c.

From this we see that dollars are changed into cents by annexing two ciphers; cents are changed into mills by annexing one cipher, and dollars into mills by annexing three ciphers.

How are dollars changed into cents? How are cents changed into mills? How are dollars changed into mills?

EXAMPLES.

1. How many cents in $6? *Ans.* 600.
2. How many mills in 13 cents? *Ans.* 130.
3. How many mills in $4 and 45 cents? *Ans.* 4450.
4. How many mills in 75 cents and 1 mill? *Ans.* 751.
5. How many cents in $9 and 13 cents? *Ans.* 913.
6. How many mills in $5 and 55 cents and 5 mills? *Ans.* 5555.

61. If we cut off one figure from the right of mills, which is dividing by 10, (Art. **30**,) the sum will be changed into cents; if from the right of cents we cut off two figures, that is, divide by 100, the sum will be changed into dollars; and if from the right of mills we cut off three figures, that is, divide by 1000, the sum will be changed into dollars. The part cut off will be a decimal portion of the denomination to which the sum is changed.

How may mills be changed to cents? How may cents be changed to dollars? How may mills be changed to dollars?

EXAMPLES.

1. How many dollars in 113 cents? *Ans.* $1·13.
2. How many dollars in 12345 mills? *Ans.* $12·345.
3. How many dollars in 1004 mills? *Ans.* $1·004.
4. How many cents in 45678 mills? *Ans.* 4567·8 cents.
5. How many dollars in 2456405 mills? *Ans.* $2456·405.

TABLE

OF SOME FRACTIONAL PARTS OF A DOLLAR FREQUENTLY USED.

5 cents $=\frac{1}{20}$ of a dollar.
6¼ cents $=\frac{1}{16}$ of a dollar.
8⅓ cents $=\frac{1}{12}$ of a dollar.
10 cents $=\frac{1}{10}$ of a dollar
12½ cents $=\frac{1}{8}$ of a dollar.
16⅔ cents $=\frac{1}{6}$ of a dollar.
20 cents $=\frac{1}{5}$ of a dollar.
25 cents $=\frac{1}{4}$ of a dollar.
33⅓ cents $=\frac{1}{3}$ of a dollar.
50 cents $=\frac{1}{2}$ of a dollar.
100 cents $=1$ dollar.

62. QUESTIONS WROUGHT BY DECIMALS.

1. Bought 4 loads of wood; the first contained 0·97 cords, the second contained 1·03 cords, the third contained

0·945 cords, the fourth contained 1·005 cords. What did the four loads measure? *Ans.* 3·95 cords.

2. In the month of May the amount of rain was 3·15 inches, in June it was 4·05 inches, in July it was 2·97 inches, and in August it was 3·03 inches. How much rain fell during these four months? *Ans.* 13·2 inches.

3. During three successive days the mean range of the barometer was 29·04 inches, 29·51 inches, and 29·73 inches respectively. What is the sum of these heights? *Ans.* 88·28 inches.

4. Bought a box of raisins for $1·75, one bushel of apples for $0·375, one cheese for $3·175, one barrel of sugar for $15·50. What did the whole amount to? *Ans.* $20·80.

5. A farmer receives $15·375 for a cow, $75 for a fine horse, $3·125 for some potatoes, $5·55 for some poultry. How much did he receive in all? *Ans.* $99·05.

6. A person bought some velvet for $3·333, some broadcloth for $18·75, some silk for 12·50, some cotton cloth for $5·405, a shawl for $12·25, some carpeting for $30·05. What did the whole amount to? *Ans.* $82·288.

7. A person borrowed $213·375, of which he has paid $107·18. How much does he still owe? *Ans.* $106·195.

8. Bought a cow for $13·25, paid $6·875. How much remains unpaid? *Ans.* $6·375.

9. What will 185 pounds of coffee cost, at $0·138 per pound? *Ans.* $25·53.

10. Bought 8·375 cords of wood, at $2·50 per cord. What did it cost? *Ans.* $20·9375.

11. What will 121·5 gallons of molasses come to, at 41 *cents per gallon?* Ans. $49·815.

12. The length of the Erie Canal is 364 miles, and it cost $7143790. What was the average expense of one mile? *Ans.* $19625·796+.

13. The Crooked Lake Canal is 8 miles long, and cost $156777. How much is this per mile? *Ans.* $19597·125.

14. In 1842, the whole number of children taught in the district schools of the State of New York was 598901; the whole amount disbursed for common schools was $1155419·90. How much was that per scholar? *Ans.* $1·929+.

15. The salary of the President of the United States is $25000. How much is that each day? *Ans.* $68·493+.

16. In one rod there are 16·5 feet. How many rods in 3573 feet? *Ans.* 216·5454+ rods.

17. Bought a farm of 137 acres for $5324. How much was that per acre? *Ans.* $38·861+.

18. If 35 miles of railroad cost $400000, how much was the average cost per mile? *Ans.* $11428·57$\frac{1}{7}$.

19. Suppose a carriage wheel to be 12 feet in circumference, how many times will it revolve in passing over a distance of 100 miles, there being 5280 feet in a mile? *Ans.* 44000 times.

20. If at each stroke of the piston rod of a locomotive engine a distance of 13·25 feet is passed over, how many strokes must be made in passing a distance of 93 miles? *Ans.* 37059·62+ times.

21. In 1845, the revenue or interest from the School Fund of the State of New York was $86828·96. During the same year there were employed 7147 teachers. If the above sum were equally divided among those teachers, what would each one receive? Ans. $12·149+.

22. In 1844, the whole number of school districts of New York was 10990, and the whole number of children in said districts, between the ages of 5 and 16 years, was 696548. What was the average number for each district?

Ans. 63·38, nearly.

23. In New York, the total number of volumes in the 11018 school district libraries was 1145250. What was the average number for each library?

Ans. 103·94+ volumes.

24. In one mile there are 1760 yards, and in one rod there are $5\frac{1}{2}=5{\cdot}5$ yards. How many rods in one mile?

Ans. 320 rods.

25. If light passes 191515 miles in a second, how many seconds will it require to pass from the sun to the earth, a distance of 95500000 miles? *Ans.* 498·65+ seconds.

26. If a cubic inch of pure water weigh 252·458 grains avoirdupois, of which 7000 make one pound, what is the weight of the Imperial or English gallon, which contains 277·274 cubic inches?

Ans. { 70000·039492 grains,
10·000005 pounds, nearly.

27. If one Imperial gallon contain 277·274 cubic inches, how many cubic inches in 8 gallons or one bushel, and how many cubic feet of 1728 inches each?

Ans. { 2218·192 cubic inches,
1·283 " feet, nearly.

28. If one cubic inch of pure water weigh 252·458 grains avoirdupois, how many grains will 1728 cubic inches, or one cubic foot, weigh, and how many pounds of 7000 grains each?

Ans. { 436247·424 grains,
62·32106 pounds, nearly.

29. A farmer sells his butter for $0·21 per pound, receiving $1613·22. How many pounds did he sell?

Ans. 7682 pounds.

30. The butter made from the milk of 53 cows during the summer, being sold for $0·20 per pound, brought $1579·40. How many pounds were sold, and what was the average produce of each cow?

Ans. { Amount sold 7897 pounds.
Average per cow 149 "

31. In a dairy of 46 cows, suppose each averages 2·5 gallons of milk daily, and that each gallon produces 1·1 pounds of cheese, how many pounds will be thus made 5·7 months of 30 days each? *Ans.* 21631·5 pounds.

32. A farmer sold as follows:

15127	pounds	of	cheese,	at	6·75	cents	per	pound,
400	"	"	butter,	"	15	"	"	"
2400	"	"	pork,	"	5	"	"	"
53	bushels	of	wheat,	"	125	"	"	"
73	"	"	barley,	"	50	"	"	"
231	"	"	corn,	"	50	"	"	"
262	"	"	oats,	"	30	"	"	"

What did the whole amount to? *Ans.* $1497·9225.

63. To find the value of articles estimated by the 100 or 1000.

What is the value of 9425 bricks, at $3·25 per 1000?

Had the price been $3·25 for each brick, we should multiply the price per brick by the number of bricks: that is, $3·25 by 9425; or, what is the same thing, we, for convenience sake, make the true multiplicand the *multiplier*, as in the following

OPERATION.
9425
3·25
47125
18850
28275
$30631·25

This value of $30631·25 is evidently 1000 times too much; therefore, to obtain the true value, we must divide it by 1000, which is done, (ART. **57**,) by removing the decimal point three places to the left; it will then become $30·63125. Had they been $3·25 per 100, then, instead of removing the decimal point three places to the left, we should have removed it two places. Hence we have this

RULE.

Multiply the number of articles by the number expressing the price per 100, *or* 1000, *and from the product cut off two of the right-hand figures when the articles are estimated by the* 100. *and three when they are estimated by the* 1000.

EXAMPLES.

1. What is the value of 1300 feet of hemlock boards, at $5·50 per 1000?

OPERATION.

1300
5·50
65000
65
$7·15000

In this product we set off five figures for decimals; two according to ART. **52**, and three more because the articles *are estimated by the* 1000.

2. What is the value of 675 feet of clear pine stuff, at $25 per 1000? *Ans.* $16·875.

3. What is the value of 11035 feet of timber, at $2·25 per 100? *Ans.* $248·2875.

4. What is the value of 90422 brick, at $3·75 per 1000? *Ans.* $339·0825.

5. What must be paid for laying 875 brick, at $3·25 per 1000? *Ans.* $2·84375.

6. A compositor worked nine months, and during that time set up at the rate of 7000 m's per day. How many thousand m's did he set up, reckoning 25 working days to the month? and how much did he receive at 15 cents per 1000 m's? *Ans.* 1575 thousand m's.
$236·25 amount he received.

7. Add together the following fractional parts of a dollar: $\frac{1}{20}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$. (See Table under Art. **61.**) *Ans.* 1.87\frac{1}{12}$.

8. A man in balancing his family accounts for one year, found his expenses as follows: for January, $98·41; for February, $81·33; for March, $102·28; for April, $125·26; for May, $74·38; for June, $73·47; for July, $65·98; for August, $87·21; for September, $70·34; for October, $122·08; for November, $79·68; for December, $52·77. His salary was $1050 per annum. What had he left at the end of the year? *Ans.* $16·81.

9. A butcher, a shoemaker, and a tailor gave orders on each other in the way of their business, and at the end of a year settled accounts. The butcher's bill against the tailor was $61·84; against the shoemaker, $39·44. The shoemaker's bill against the butcher was $24 30; against the tailor, $19·15. The tailor's bill against the butcher

was $42·07; against the shoemaker, $39·39. Who received balances in cash?

Ans. Butcher received from tailor, $19·77.
" " " shoemaker, $15·14.
Tailor " " " $20·24.

NOTE.—By an Act of Congress passed Feb. 20, 1849, the *Double Eagle* and the *Gold Dollar* were added to the Gold Coins of the United States. The act directing the coinage of these pieces is to be in force until March 4, 1851. See Art. 58.

DENOMINATE NUMBERS.

64. A SIMPLE NUMBER is an expression for a certain number of units having no reference to particular things. Thus 37 is the same as 37 times one, abstractly considered; that is, considered apart from anything that units might represent. It does not mean 37 times a pound, foot, dollar, or anything else.

A DENOMINATE NUMBER is an expression for a certain number of units having reference to particular things. It *denominates* what things are meant. Thus 8 yards is a denominate number whose unit is 1 yard; 8 dollars is a denominate number whose unit is one dollar.

Several numbers of different denominations are frequently grouped together, as 6 feet 3 inches.

All our different kinds of weights and measures are denominate numbers. It is much to be regretted that we are obliged to employ such a variety of different measures, when the same end would be accomplished by one measure for weight, and one for each of the three geometrical *magnitudes, lengths, surfaces* and solids, and one for time

The French government have adopted such a system of weights and measures, graduated on the decimal scale of notation.

In multiplication, the multiplicand being repeated a certain number of times, or a certain fraction of a time, when the multiplier is a fraction, it follows that the multiplier, considered as a multiplier, must always be regarded as a simple or abstract number. And since the product is a repetition of the multiplicand, it must be like the multiplicand; that is, if the multiplicand is an abstract number, the product will be an abstract number; if the multiplicand is a denominate number, the product will be a denominate number of the same kind.

In division, the quotient showing how many times the divisor is contained in the dividend, or what fraction of a time when the divisor is greater than the dividend, it follows that the quotient must be regarded as an abstract number; and that the divisor and dividend must be alike.

NOTE.—In many cases, however, the process of division is rather the dividing of a dividend *into as many equal parts as are indicated by the divisor;* in which case, the quotient, expressing the units in one of those parts, is of the same kind as the dividend, while the divisor is to be regarded as an abstract number. See Example, ART. **86.**

What is a simple number? What is a denominate number? What kind of numbers are all our different weights and measures? What is said of the French measures? In multiplication, can the multiplier ever be a denominate number? Are the product and multiplicand always alike? What is said of the quotient? What is said in the note?

The following are some of the most important tables of weights and measures at present employed in this country.

ENGLISH MONEY.

65. The denomination of English money are *Farthings, Pence,* Shillings, and Pounds.

The pound sterling, which was not a coin, but a bank note of 20 shillings, has now gone into disuse, and a gold coin, called a *Sovereign*, supplies its place; but the name pound is still given to 20 shillings.

TABLE.*

4 farthings, *far.* make 1 penny, *d.*
12 pence " 1 shilling, *s.*
20 shillings " 1 pound, £.

far. *d.*
4= 1 *s.*
48= 12= 1 £
960=240=20=1

NOTE.—Farthings are sometimes expressed in fractions of a penny, as follows: 1 farthing=¼ *d.*, 2 farthings=½ *d.*, 3 farthings =¾ *d.*

What are the denominations of English money? Which denomination is never coined? What gold coin is equivalent in value to one pound? Repeat the Table. How are farthings sometimes expressed?

TROY WEIGHT.

66. The original of all weights used in England was a grain or corn of wheat, gathered out of the middle of the ear; 32 of these, well dried, were to make one pennyweight, 20 pennyweights one ounce, and 12 ounces one pound. But in latter times, it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in common use.

Coins, precious metals, jewels, and liquors, are weighed by Troy weight.

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[*] The full weight and value of English gold and silver coin are as in the succeeding *table, note.*

TABLE.*

24 grains *gr.* make 1 pennyweight, *pwt.*
20 pennyweights " 1 ounce, *oz.*
12 ounces " 1 pound, *lb.*

gr.	*pwt.*		
24=	1	*oz.*	
480=	20=	1	*lb.*
5760=	240=	12=	1

What was the original of all weights used in England? How was the grain obtained? Is it still used as a weight? What substances are weighed by Troy weight? Repeat the Table.

APOTHECARIES' WEIGHT.

67. This weight, as its name would imply, is used in weighing medicines in small quantities, as for prescriptions. But drugs and medicines in gross are bought and sold by Avoirdupois Weight. The pound and ounce Apothecaries' Weight are the same as in Troy Weight.

	Name of Coin.	Value. £	Value. s.	Value. d.	Weight. pwt.	Weight. gr.
Gold.	A guinea,	1	1	0	5	$9\frac{1}{2}$
	Half guinea,	0	10	6	2	$16\frac{3}{4}$
	Quarter guinea,	0	5	3	1	$8\frac{1}{4}$
	Sovereign,	1	0	0	5	$3\frac{3}{11}$
	Half sovereign,	0	10	0	2	$13\frac{7}{11}$
Silver.	A crown,	0	5	0	18	$4\frac{4}{11}$
	Half crown,	0	2	6	9	$2\frac{2}{11}$
	Shilling,	0	1	0	3	$15\frac{3}{11}$
	Sixpence,	0	0	6	1	$19\frac{7}{11}$

---

* This scale of weights is said to have been borrowed originally from *Troyes* in France—hence its name. Some, however, contend that the name has reference to *the monkish title given* to London, of *Troy Novant*.

TABLE.

20	grains *gr.*	make	1	scruple,	℈
3	scruples	"	1	dram,	ʒ
8	drams	"	1	ounce,	℥
12	ounces	"	1	pound,	℔

gr.	℈	ʒ	℥	℔
20=	1			
60=	3=	1		
480=	24=	8=	1	
5760=	288=	96=	12=	1

For what purpose is Apothecaries' Weight used? Do its pound and ounce differ from Troy Weight?

AVOIRDUPOIS WEIGHT.

68. By this weight are weighed all things of a coarse or drossy nature, as bread, butter, cheese, flesh, groceries and some liquids; all metals, except gold and silver.

TABLE.

16	drams *dr.*	make	1	ounce,	*oz.*
16	ounces	"	1	pound,	*lb.*
28	pounds	"	1	quarter,	*qr.*
4	quarters	"	1	hundred weight,	*cwt.*
20	hundred weight	"	1	ton,	*T.*

dr.	*oz.*	*lb.*	*qr.*	*cwt.*	*T.*
16=	1				
256=	16=	1			
7168=	448=	28=	1		
28672=	1792=	112=	4=	1	
573440=	35840=	2240=	80=	20=	1

The pound Avoirdupois contains 7000 grains.

From the preceding table, it will be seen that 112 pounds instead of 100, are called one hundred weight. In most cases however the hundred weight is taken equal to 100 pounds, and 25 pounds, instead of 28, is called a quarter. Coal merchants in buying coal receive 112 pounds for a hundred weight, and 20 hundred weight for a ton, making 2240 pounds; but they retail it at 2000 pounds for a ton.

What substances are weighed by Avoirdupois Weight? Repeat the Table. By this weight how many pounds make one hundred weight? In most cases how many pounds make a hundred weight? How is coal usually bought and sold?

LONG MEASURE.

69. It is usual, at the present time, to derive the measure of length from that of a pendulum vibrating once in a second of time. The length of such pendulums will vary for different latitudes, as here given.

PLACES.	LATITUDES.			LENGTH IN INCHES.
Equator,	0°	0′	0″	39·01612
Cape of G. H.,	33	55	15	39·07815
New York,	40	42	43	39·10120
Paris,	48	50	14	39·12929
London,	51	31	8	39·13929
The Pole,	90	0	0	39·21820

The French government derive their linear unit of measure from one quarter of the circumference of a great circle of the earth passing through the poles. Having determined by actual surveys the length of that portion of such a circle comprised between the parallels of Dunkirk and Barcelona, they deduced its entire length from the equator to the pole, and took one ten millionth part of it for a *metre*. This method gave for the French metre 39·37079 English inches, equal 3·2809 feet, nearly.

TABLE.

3	barleycorns *bar.*[*]	make	1 inch,	*in.*
12	inches	"	1 foot,	*ft.*
3	feet	"	1 yard,	*yd.*
5½	yards	"	1 rod, perch, or pole,	*rd.*
40	rods	"	1 furlong,	*fur.*
8	furlongs	"	1 mile,	*mi.*
3	miles	"	1 league,	*L.*
†69⅙	miles, nearly	"	1 degree,	*deg.* or °.

in. *ft.*
12 = 1 *yd.*
36 = 3 = 1 *rd.*
198 = 16½ = 5½ = 1 *fur.*
7920 = 660 = 220 = 40 = 1 *mi.*
63360 = 5280 = 1760 = 320 = 8 = 1

From what is the measure of length, at the present time, usually derived? Mention the lengths of the second's pendulum for the places given above. How do the French derive their measure of length? How is their metre obtained? What is its length in English inches? What is the length in feet?

Repeat the Table of Long Measure.

---

[*] This measure has fallen into disuse, and for small portions of an inch, we use one-eighth, one-tenth, and one-sixteenth.

† The latest measurements give the equatorial diameter of the earth equal to 7925·648 miles, and its circumference equal to 24899 miles, which, divided by 360, gives the length of a degree 69⅙ miles, nearly. The circumference corresponding with the equator is nearly circular, while the circumference passing through the poles is elliptical.

70. ## CLOTH MEASURE.

TABLE.

2¼	inches *in.*	make	1 nail,	*na.*
4	nails	"	1 quarter of a yard,	*qr.*
3	quarters	"	1 Ell Flemish,	*E. Fl.*
4	quarters	"	1 yard,	*yd.*
4	*qr.* 1¼ *in.*	"	1 Ell Scotch,	*E. S.*
5	quarters	"	1 Ell English,	*E. E.*
6	quarters	"	1 Ell French,	*E. Fr.*

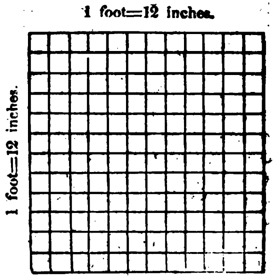
Repeat the Table of Cloth Measure.

SQUARE MEASURE.

71. This measure is used for estimating artificers' work, such as boards, glass, pavements, plastering, flooring, painting, and any other kind of work where length and breadth only are concerned. It is always employed for measuring land, and for this reason it is sometimes called *Land Measure.*

A square is a figure having four equal sides, and all its angles *right*, that is, the sides are perpendicular to each other.

If the length of one of the sides is one inch, it is called a *square inch.* If the length of one of the sides is one foot, or 12 inches, it is called a *square foot*, which by the adjacent figure we see is composed of 12 × 12 = 144 *square inches.*



In a similar manner, if we

had a square, each of whose sides was 3 feet, then it would contain 3 × 3 = 9 *Sq. feet*, which is called one ***square yard***, since 3 feet = 1 yard.

TABLE.

144	square inches	*Sq. in.*	make	1	square foot,	*Sq. ft.*
9	square feet		"	1	square yard,	*Sq. yd.*
30¼	square yards		"	1	square rod or pole,	*P.*
40	square poles		"	1	rood,	*R.*
4	roods		"	1	acre,	*A.*
*640	acres		"	1	square mile.	*M.*

Sq. in. *Sq. ft.*
144 = 1 *Sq. yd.*
1296 = 9 = 1 *P.*
39204 = 272¼ = 30¼ = 1 *R.*
1568160 = 10890 = 1210 = 40 = 1 *A.*
6272640 = 43560 = 4840 = 160 = 4 = 1

In measuring land, Gunter's chain is used; its length is 4 rods, or 66 feet. It is divided into 100 links.

$7\frac{92}{100}$	inches	make	1	link,	*l.*
100	links, or 4 rods, or 66 feet,	"	1	chain,	*c.*
80	chains	"	1	mile,	*m.*
10000	square links	"	1	square chain.	
10	square chains	"	1	acre,	*A.*

What use is made of Square Measure? When employed in measuring land, how is it called? What is a square? When a square is one inch on each side how is it called? When it is one foot or 12 inches on each side how is it called? When it is one yard on each side how is it called? Repeat the Table of Square Measure. In Land Measure, with what are the sides of the field usually measured? How long is this chain? Repeat the Table of Land Measure.

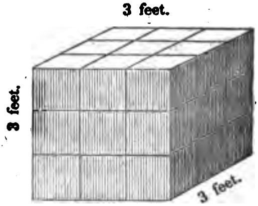
*** The acre is in all cases applied to surface or area. There is no such thing as an acre long, or a cubic acre. It is of such a magnitude as not to admit of being *accurately given in the form of* a square, having the sides exactly determined. The *same remarks are applicable* to the rood.**

SOLID, OR CUBIC MEASURE.

72. This is used in measuring all bodies where we have regard to length, breadth, and thickness, such as earth, stone, timber, &c.

A Cube is a solid bounded by six equal squares, resembling a common tea-chest.

If the sides of a cube are each one inch long, it is called a *cubic inch*. If each side is one foot, it is called a *cubic foot*. If a side is one rod, it is called a *cubic rod*.



In the adjoining figure we have endeavored to represent a cube, each side of which is 3 feet, or one yard, and consequently it is one *cubic* yard.

The top, which is equal to the base, contains $3 \times 3 = 9$ square feet; hence, if this was only one foot in height, it would contain 9 cubic feet; but as it is 3 feet in height, it must therefore contain 3 times $9 = 27$ cubic feet. Hence, one cubic yard is equivalent to $3 \times 3 \times 3 = 27$ cubic feet.

In the same way one cubic foot is equivalent to $12 \times 12 \times 12 = 1728$ cubic inches.

TABLE.

1728	solid inches	*S. in.*	make	1 solid foot,	*S. ft.*
27	solid feet		"	1 solid yard,	*S. yd.*
*40	feet of round timber or 50 feet of hewn timber		"	1 ton,	*Ton.*
128	solid feet		"	1 cord of wood,	*C.*

---

* *A ton of round timber* is so much as, when hewed, shall make 40 cubic feet.

A pile of wood 4 feet wide, 4 feet high, and 8 feet long, will make one cord. One foot in length of such a pile is sometimes called *a cord foot.* It contains 16 solid feet; consequently 8 cord feet make one cord.

For what is Solid Measure used? What is a Cube? In a cubic yard how many cubic feet? In a cubic foot how many cubic inches? How many cubic feet of round timber make a ton? How many of hewn timber? How many cubic feet make a cord of wood? Explain what is meant by a cord foot.

WINE MEASURE.

73. By this are measured all liquids except beer.

TABLE.

4	gills *gi.*	make	1	pint,	*pt.*
2	pints	"	1	quart,	*qt.*
4	quarts	"	1	gallon,	*gal.*
31½	gallons	"	1	barrel,	*bar.*
63	gallons	"	1	hogshead,	*hhd.*
2	hogsheads	"	1	pipe,	*pi.*
2	pipes	"	1	tun,	*tun.*

gi. *pt.*
4 = 1 *qt.*
8 = 2 = 1 *gal.*
32 = 8 = 4 = 1 *bar.*
1008 = 252 = 126 = 31½ = 1 *hhd.*
2016 = 504 = 252 = 63 = 2 = 1 *pi.*
4032 = 1008 = 504 = 126 = 4 = 2 = 1 *tun.*
8064 = 2016 = 1008 = 252 = 8 = 4 = 2 = 1

The wine gallon contains 231 cubic or solid inches.

What liquids are measured by Wine Measure? Repeat the Table. How many *cubic inches in the wine gallon?*

74. ALE, OR BEER MEASURE.

TABLE.

2	pints *pt.*	make	1 quart,	*qt.*
4	quarts	"	1 gallon,	*gal.*
36	gallons	"	1 barrel,	*bar.*
1½	barrels	"	1 hogshead,	*hhd.*

pt. *qt.*
2= 1 *gal.*
8= 4= 1 *bar.*
288=144=36=1 *hhd.*
432=216=54=1½=1

The beer gallon contains 282 cubic or solid inches.

What is measured by Beer Measure? Repeat the Table. How many cubic inches in the beer gallon?

DRY MEASURE.

75. By this are measured all dry wares, as grain, seeds, roots, fruits, salt, coal, sand, oysters, &c.

TABLE.

2	pints *pt.*	make	1 quart,	*qt.*
8	quarts	"	1 peck,	*pk.*
4	pecks	"	1 bushel,	*bu.*
*36	bushels	"	1 chaldron,	*ch.*

pt. *qt.*
2= 1 *pk.*
16= 8= 1 *bu.*
64= 32= 4= 1 *ch.*
2304=1152=144=36=1

---

* In the United States 32 bushels=1 chaldron.

By the English statute the dry gallon must contain 268⅘ cubic or solid inches. The corn or Winchester bushel must contain 2150⅖ cubic or solid inches. This measure is of a cylindric form, 8 inches deep and 18½ inches in diameter.

By an act of Parliament, which took effect the 1st of January, 1826, the imperial gallon of 277·274 cubic inches was adopted as the only gallon. This gallon was to consist of 10 pounds, avoirdupois, of distilled water.

NOTE.—If we divide 1728, the number of cubic inches in one cubic foot, by 277·274, the number of cubic inches in the gallon, we shall obtain 6·2321 for a quotient, which is the number of gallons in one cubic foot. Multiplying 6·2321 by 10, the number of pounds in one gallon, we obtain 62·321 for the number of avoirdupois pounds in one cubic foot of water. In one avoirdupois pound there are 7000 grains, and in 10 pounds there are 70000 grains. But 10 pounds is the weight of one gallon, which contains 277·274 cubic inches. Hence, dividing 70000 by 277·274, we find 252·458, the weight in grains of one cubic inch of water.

According to the Revised Statutes of the state of New York, a cubic foot of distilled water, when estimated under prescribed circumstances, is to consist of 62½ pounds, or 1000 ounces avoirdupois weight. Eight pounds of such water is to constitute the gallon for liquid measure, and ten pounds is to make the gallon for dry measure.

What articles are measured by Dry Measure? Repeat the Table. How many cubic inches in the dry gallon, according to the English statute? How many cubic inches in a bushel? What is the form and dimensions of the Winchester bushel measure? How many cubic inches in the English imperial gallon? The imperial gallon contains how many pounds of distilled water? One cubic foot of water weighs how many avoirdupois pounds? How many Troy pounds? One cubic inch of water weighs how many grains? How many pounds of water constitute the dry gallon, according to the Revised Statutes of New York? How many pounds make the liquid gallon?

76. TIME.

TABLE.

60 seconds	*sec.*	make	1 minute,	*min.*
60 minutes		"	1 hour,	*hr.*
24 hours		"	1 day,	*da.*
7 days		"	1 week,	*wk.*
4 weeks		"	1 month,	*mo.*
13 mo., 1 da., 6 hr., or 365 da., 6 hr.		"	1 Julian year,	*yr*

sec.	*min.*	*hr.*	*da.*	*wk.*	*yr.*
60=	1				
3600=	60=	1			
86400=	1440=	24=	1		
604800=	10080=	168=	7 =	1	
31557600=	525960=	8766=	$365\frac{1}{4}$=	$52\frac{5}{28}$=	1

The true length of the solar year is 365·242217 days, or about 365 da. 5 hr. 48 m. $47\frac{1}{2}$ sec.

The civil year is also divided into 12 calendar months, as follows:

			DAYS.				DAYS.
1 month,		January,	31	7 month,		July,	31
2	"	February,	28 or 29	8	"	August,	31
3	"	March,	31	9	"	September,	30
4	"	April,	30	10	"	October,	31
5	"	May,	31	11	"	November,	30
6	"	June,	30	12	"	December,	31
							365 or 366

If the year exceeded 365 days by 6 hours exactly, then once in four years these hours would amount to another day. Hence, once in four years, an additional day is given *to the month* of February; and such years are called

Bissextile or Leap years. But, since this excess is not quite 6 hours, this rule of adding one day to February every fourth year is interrupted, and the centennial years, which are not divisible by 400, are regarded as common years.*

Hence, any year, (except a centennial year,) which is divisible by 4, is a Leap year, or consists of 366 days.

Centennial years which are divisible by 400 are regarded as Leap years; all others are considered as common years.

1796, 1804, 1808, 1812, 1816, 1820, 1824, 1828, 1832, 1836, 1840, were all Leap years. 1800, not being divisible by 400, was a common year of 365 days; the same may be said of 1900; but the year 2000, being divisible by 400, will be a Leap year.

The number of days in the respective months may be recalled by recollecting the following versification:

Thirty days hath September,
April, June, and November—
All the rest have thirty-one,
Excepting February alone,
Which has but twenty-eight in fine,
Till Leap Year gives it twenty-nine.

Repeat the Table for Time. What is the length of the solar year to the nearest second? What is the more accurate value in decimals? Into how many calendar months is the civil year divided? Repeat their names and the number of days belonging to each. How often in general is an additional day added to February? What are such years styled? Is the rule of counting every fourth year Leap year correct? Are centennial years, which are not divisible by 400, Leap years? Was 1800 a Leap year? Mention the next preceding and next following Leap year to 1800.

It is very desirable to be able readily to determine the number of days from any particular date to any other date. For this purpose, we will give the following

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[*] There is still a further modification which takes place at the end of every 1900 years, which *it is unnecessary* to explain in this place.

TABLE,

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Ap'l	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY,	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY,	334	365	28	59	89	120	150	181	212	242	273	303
MARCH,	306	337	365	31	61	92	122	153	184	214	245	275
APRIL,	275	306	334	365	30	61	91	122	153	183	214	244
MAY,	245	276	304	335	365	31	61	92	123	153	184	214
JUNE,	214	245	273	304	334	365	30	61	92	122	153	183
JULY,	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST,	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER,	122	153	181	212	242	273	303	334	365	30	61	91
OCTOBER,	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER,	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER,	31	62	90	121	151	182	212	243	274	304	335	365

As an example, suppose we wish the number of days from November 6th to the 15th of next April. We find November in the left-hand vertical column, and April at the top line of the table, and at the intersection we find 151 days. So that from November 6th to April 6th is 151 days; consequently, adding 9, we find 160 for the number of days between November 6th and April 15th.

This table is constructed on the supposition of 28 days to February. When there are 29 days in February the proper allowance must be made.

EXAMPLES.

1. How many days from May 3d to the 4th of the next *July?* Ans. 62 days.

2. How many days from July 4th to the 25th of the xt December? *Ans.* 174 days.

3. How many days from March 21st to the 23d of the xt September? *Ans.* 186 days.

4. How many days from September 23d to the 21st of e next March? *Ans.* 179 days.

5. How many days from June 21st to the 22d of the xt December? *Ans.* 184 days.

6. How many days from December 22d to the 21st of e next June? *Ans.* 181 days.

7. How many days from March 21st to the 21st of the xt June? *Ans.* 92 days.

8. How many days from Jan. 13th, 1848, to September th of the same year? *Ans.* 248 days.

CIRCULAR MEASURE, OR MOTION.

77. By this is estimated Latitude and Longitude, and e motion of the heavenly bodies which appear to move circles. Every circle, whether great or small, is divided to 360 degrees.

TABLE.

60 seconds ″	make	1 minute,	′
60 minutes	"	1 degree,	°
30 degrees	"	1 sign,	s.
12 signs or 360°	"	1 circle,	cr.

″ ′

60 = 1 °

3600 = 60 = 1 s.

108000 = 1800 = 30 = 1 cr.

1296000 = 21600 = 360 = 12 = 1

The sun appears to pass completely around the earth in 24 hours, that is, it appears to move westward over 360° of longitude in 24 hours. Consequently, in one hour it will move over $\frac{1}{24}$ of 360° = 15° of longitude. Hence, if the difference in the longitudes of two places is 15°, it will be noon at the more easterly place, just one hour before it is noon at the other place. And in all cases, the difference in time of any two places will be at the rate of one hour for every 15° of longitude between the two places. As an example, suppose the city of Washington to be 77° west of Greenwich: it is required to find what time it is at Washington, when it is noon at Greenwich.

Dividing 77° by 15° we have 5 $\frac{2}{15}$ for the number of hours difference in time, that is, 5h. 8m. And as the apparent motion of the sun is westward, it must be earlier at Washington than at Greenwich. Therefore, when it is noon at Greenwich, it is 5h. 8m. before noon at Washington; that is, it is at Washington 6h. 52m. A. M.

What use is made of Circular Motion? Into how many degrees are all circles supposed to be divided? Repeat the Table. Over how many degrees of longitude does the sun appear to move in 24 hours? Over how many degrees in 1 hour? What is the difference of time corresponding to 77°? When it is noon at Greenwich, what time is it at Washington, 77° west of Greenwich?

78. Measures, &c., not included in the foregoing tables.

6 points	make	1 line	used in measuring length of clock pendulum rods.
12 lines	"	1 inch	
4 inches	"	1 hand	used in measuring the height of horses.
6 feet	"	1 fathom	used in measuring depths at sea.

12	individual things	make	1 dozen.
12	dozen	"	1 gross.
12	gross	"	1 great gross.
20	individual things	"	1 score.
24	sheets of paper	"	1 quire.
20	quires	"	1 ream.
112	pounds	"	1 quintal of fish.
200	"	"	1 barrel of pork or beef.
196	"	"	1 barrel of flour.

Repeat the above tables.

BOOKS.

79. A sheet folded into two leaves is called a folio.
" folded into four leaves is called a quarto, or 4to.
" folded into eight leaves is called an octavo, or 8vo.
" folded into twelve leaves is called a duodecimo, or 12mo.
" folded into eighteen leaves is called an 18mo.

When a sheet is folded into two leaves what is it called? How called when folded into four leaves? How, when folded into eight leaves? How, when folded into twelve leaves? How, when folded into eighteen leaves?

REDUCTION.

80. REDUCTION is the changing of numbers from one name or denomination to another, without altering their *value.*

When the denominations are to be reduced from a higher denomination to a lower, it is called *Reduction Descending;* but when they are to be reduced from a lower to a higher denomination, it is called *Reduction Ascending.*

REDUCTION DESCENDING.

Let it be required to reduce £7 5*s*. 10*d*. 3*far*. to farthings.

OPERATION.

	7	the number of pounds.
Multiply by	20,	the number of shillings in one pound.
	140	product in shillings.
Add	5	shillings.
	145	the number of shillings.
Multiply by	12,	the number of pence in one shilling.
	290	
	145	
	1740	product in pence.
Add	10	pence.
	1750	the number of pence.
Multiply by	4,	the number of farthings in one penny.
	7000	product in farthings.
Add	3	farthings.
	7003	the number of farthings sought.

From the above operation, we readily deduce this general

RULE.

Multiply the number in the highest denomination by the number indicating how many of the next lower make one in *that higher; to this product add the number, if any, belonging to this lower denomination; we shall thus obtain an equivalent value in the next lower denomination.*

II. Proceed in a similar way for all the successive denominations; the last result will be the number sought.

What is Reduction? When is it called Descending? And when Ascending? Repeat the rule for Reduction Descending.

REDUCTION ASCENDING.

81. Let it be required to reverse the last example, that is, to find the number of pounds, shillings, pence, and farthings, in 7003 farthings.

We must obviously perform a reverse operation to that performed under Reduction Descending.

OPERATION.

far.
4)7003
1750*d.* 3*far.* remainder.

d. *s.*
12)1750(145
12
55
48
70
60
10*d.* remainder.

s.
2|0)14|5
£7 5*s.* remainder.

Collecting results, we have 7003 farthings, equivalent to £7 5*s.* 10*d.* 3*far.*

EXPLANATION.

First, we divide the number of farthings, 7003, by 4, because 4 farthings make one penny; the quotient is 1750 pence, and 3 farthings remaining.

Secondly, we divide the number of pence, 1750, by 12, because 12 pence make one shilling; the work being performed by Long Division, we get for the quotient 145 shillings, and 10 pence remaining.

Thirdly, we divide the number of shillings, 145, by 20, because 20 shillings make one pound; cutting off the cipher from the right of 20, and the right-hand figure from the dividend, (ART. **30**,) we perform the work by Short Division, and obtain the quotient, 7 pounds, and 5 shillings remaining.

We may, therefore, deduce this general

RULE.

I. Divide the given number by as many of its denomination as make one of the next higher; write down the quotient and remainder, if any.

II. Divide the quotient by as many of its denomination as make one of the next higher; write this new quotient and the remainder as before.

III. Proceed in this way through all the denominations to the highest, and the quotient last found, together with the several remainders, if any, will give the value sought.

Repeat the Rule for Reduction Ascending.

EXAMPLES.

1. In £47 5*s.* 2*d.* 1 *far.*, how many farthings?

OPERATION.

```
  £47   5s.   2d.   1 far.
    20
   ---
   945 shillings.
    12
  ----
  1892
  945
 -----
 11342 pence.
     4
 -----
 45369 farthings.
```

2. In 118567 farthings, how many pounds, shillings, pence, and farthings?

OPERATION.

```
        far.
     4)118567
     --------
        29641  3 farthings.
         d.     s.
  12)29641(2470
     24
     --
      56
      48
      --
       84
       84
       --
        1  penny.
     2|0)247|0
     ---------
        £123  10 shillings.
```

Hence, 118567 farthings are equal to £123 10*s.* 1*d.* 3*far.*

3. Reduce £75 to shillings. *Ans.* 1500*s.*

4. Reduce 19*s.* 6*d.* to pence. *Ans.* 234*d.*

5. Reduce 15*s.* 3*d.* 2*far.* to farthings. *Ans.* 734*far.*

6. In 48926 grains, Troy Weight, how many pounds, ounces, pennyweights, and grains? *Ans.* 8*lb.* 5*oz.* 18*pwt.* 14*gr.*

7. In 3605 pennyweights, how many pounds, ounces, and pennyweights? *Ans.* 15*lb.* 0*oz.* 5*pwt.*

8. In 1000 ounces, Troy Weight, how many pounds and ounces? *Ans.* 83*lb.* 4*oz.*

9. In 4*lb.* 6*oz.* 13*pwt.* 5*gr.* how many grains? *Ans.* 26237*gr.*

10. In 100*lb.* 1*gr.* how many grains? *Ans.* 576001*gr.*

11. In 4 ℔ 5 ℥ 1 ʒ how many drams? *Ans.* 425 ʒ.

12. In 1000 grains, Apothecaries' Weight, how many ounces, drams, scruples, and grains? *Ans.* 2 ℥ 0 ʒ 2 ℈.

13. In 11521 grains, Apothecaries' Weight, how many pounds? *Ans.* 2 ℔ 0 ℥ 0 ʒ 0 ℈ 1*gr.*

14. In 873450 drams, Avoirdupois Weight, how many tons? *Ans.* 1*T.* 10*cwt.* 1*qr.* 23*lb.* 14*oz.* 10*dr.*

15. Reduce 5*cwt.* 21*lb.* 4*oz.* to ounces. *Ans.* 9300 ounces.

16. Reduce 1*T.* 1*cwt.* 1*dr.* to drams. *Ans.* 602113 drams.

17. Reduce 856702 drams to tons. *Ans.* 1*T.* 9*cwt.* 3*qr.* 14*lb.* 7*oz.* 14*dr.*

18. In 4355 inches, how many yards? *Ans.* 120*yds.* 2*ft.* 11*in.*

19. In 248 miles, how many inches? *Ans.* 15713280 inches.

20. How many inches in 360 degrees of $69\frac{1}{6}$ miles to each degree, which is the circumference of the earth, nearly. *Ans.* 1577664000 inches.

21. In 12121212 barleycorns, how many miles?
Ans. 63*mi.* 6*fur.* 6*rd.* 0*yd.* 1*ft.* 4*in.*

22. Reduce 12 Ells French to nails. *Ans.* 288 nails.

23. Reduce 11 Ells English, 3 quarters, to quarters.
Ans. 58 quarters.

24. Reduce 10 Ells Flemish, 3 quarters, 1 nail, to nails.
Ans. 133 nails.

25. Reduce 4 yards to quarters. *Ans.* 16 quarters.

26. In 1000 nails, how many yards? *Ans.* 62*yds.* 2*qr.*

27. How many inches in 6 yards, 3 quarters?
Ans. 243 inches.

28. How many square inches in 10 square feet?
Ans. 1440 square inches.

29. In 3 square miles, how many square rods or poles?
Ans. 307200*P.*

30. In 3 acres, 27 rods, how many square feet?
Ans. $138030\frac{3}{4}$ square feet.

31. In 26025 square feet, how many square roods?
Ans. 2*R.* 15*P.* $161\frac{1}{4}$ *sq. ft.*

32. In 70000 square links, how many square chains?
Ans. 7 square chains.

33. How many square links in 5 acres?
Ans. 500000 square links.

34. In 17 cords of wood, how many cubic feet?
Ans. 2176 cubic feet.

35. In 17 tons of round timber, how many cubic inches?
Ans. 1175040 cubic inches.

36. Reduce 17900345 cubic inches to tons of hewn timber. *Ans.* 207 Tons, 8 cubic feet, 1721 cubic inches.

37. In 1000 cord feet of wood, how many cords?
Ans. 125 cords

38. In 19 cubic feet, how many cubic inches?
Ans. 32832 cubic inches

39. In 16 hogsheads of wine, how many gills?
Ans. 32256 gills.

40. In 10000 gills of wine, how many barrels?
Ans. 9 barrels 29 gallons.

41. Reduce 2 pipes, 7 barrels, 3 quarts of wine, to pints.
Ans. 3786 pints.

42. Reduce 31752 gills of wine to barrels.
Ans. 31 barrels, 15 gallons, 3 quarts

43. Reduce 201600 gills to tuns of wine.
Ans. 25 tuns.

44. Reduce 11 hogsheads of beer to pints.
Ans. 4752 pints.

45. In 100000 pints of beer, how many hogsheads?
Ans. 231 hogsheads, 26 gallons.

46. In 10 hogsheads, 1 quart, 1 pint of beer, how many pints? *Ans.* 4323 pints.

47. In 36 bushels how many pints? *Ans.* 2304 pints.

48. In 25 chaldrons 29 bushels, how many quarts?
Ans. 29728 quarts.

49. In 10000 pints, how many chaldrons?
Ans. 4*ch.* 12*bu.* 1*pk.*

50. In 1597 quarts, how many bushels?
Ans. 49*bu.* 3*pk.* 5*qt.*

51. In 30 days, how many seconds? *Ans.* 2592000*sec.*

52. In 19 years of $365\frac{1}{4}$ days each, how many hours?
Ans. 166554 hours.

53. In 25 years 6 days, how many seconds?
Ans. 789458400 seconds.

54. How many days from the birth of Christ to Christmas, 1843, allowing the years to consist of 365 days 6 hours? *Ans.* 673155 days 18 hours.

55. A person was born May 3, 1795. How many days old was he May 3, 1821, paying particular attention to the order of leap year? *Ans.* 9496 days.

56. Suppose a person was born February 29, 1796; how many birthdays will he have seen on February 29, 1844, not counting the day on which he was born?*

Ans. 11 birth-days.

57. In 3 signs 18 degrees, how many seconds?

Ans. 388800″.

58. In 6 signs 9 degrees, how many degrees?

Ans. 189°.

59. In 1000′ how many degrees? *Ans.* 16° 40′.

60. In 10000″ how many degrees? *Ans.* 2° 46′ 40″.

61. Reduce 45° 45′ 35″ to seconds. *Ans.* 164735″.

62. In 1000 things, how many dozen?

Ans. 83 dozen and 4 over.

63. How many buttons in 6⅓ dozen?

Ans. 76 buttons.

64. In 80000 tacks, how many gross?

Ans. 555 gross, 6 dozen and 8.

65. In three score and ten years, how many years?

Ans. 70 years.

66. In 15 quires of paper, how many sheets?

Ans. 360 sheets.

67. In a ream of paper, how many sheets?

Ans. 480 sheets.

---

* It must be recollected that the year 1800 was a common year, having no 29th of February.

ADDITION OF DENOMINATE NUMBERS.

82. If we wish to find the sum of £6 5*s*. 3*d*. 1 *far*., £7 1*s*. 10*d*. 2 *far*., £1 13*s*. 5*d*., £4 18*s*. 0*d*. 2 *far*., we proceed as follows:

OPERATION.

£	*s*.	*d*.	*far*.
6	5	3	1
7	1	10	2
1	13	5	0
4	18	0	2
£19	18*s*.	7*d*.	1 *far*.

Placing the numbers of the same denomination directly under each other, we add up the column of farthings, which we find to be 5. But we know that 5 farthings are equivalent to 1 penny and 1 farthing; we therefore write down the 1 farthing under the column of farthings, and carry the penny into the next column, whose sum thus becomes 19 pence, which is the same as 1 shilling and 7 pence; we write down the 7 pence under the column of pence, and carry the shilling to the column of shillings; whose sum then becomes 38 shillings, which is the same as 1 pound and 18 shillings; we write down the 18 shillings under the column of shillings, and carry the pound into the column of pounds, whose sum then becomes 19 pounds; and since pounds is the highest denomination, we write down the whole.

From this example we may deduce this general

RULE.

I. Place the numbers so that those of the same donomination may stand directly under each other, and draw a line beneath them.

II. Add the numbers in the lowest denomination; divide their sum by the number expressing how many it takes of such denomination to make one of the next higher Write *the remainder under the column added, and carry the quotient to the next column; which add as before.*

III. Proceed thus through all the denominations to the highest, whose sum must be set down entire.

How do you place denominate numbers which are to be added? Which do you first add? Having added the column of lowest denominations, explain the subsequent work.

EXAMPLES.

£	s.	d.
7	13	3
3	5	10½
6	18	7
0	2	5¾
4	0	3
17	15	4½
39	15	9¾

£	s.	d.
11	0	5½
2	4	4
0	5	6¼
1	3	4
10	10	10
25	4	5¾

£	s.	d.
5	5	5
8	1	7¾
2	0	1½
13	0	11¾
6	6	6
34	14	8

TROY WEIGHT.

lb.	*oz.*	*pwt.*	*gr.*
10	10	10	10
0	2	0	23
3	0	17	0
2	2	1	0
1	0	2	20
17	3	12	5

lb.	*oz.*	*pwt.*	*gr.*
6	5	4	1
1	11	19	13
0	3	0	4
8	9	1	2
4	4	19	0
21	10	3	20

lb.	*oz.*	*pwt.*	*gr.*
7	3	0	5
11	2	17	22
40	0	0	20
2	10	15	17
0	6	18	16
61	11	13	8

APOTHECARIES' WEIGHT.

℔	℥	ʒ	℈	*gr.*
8	10	7	2	19
10	0	6	0	10
0	1	2	1	15
5	1	2	1	15
8	0	5	1	13
32	*3*	*0*	*2*	12

℔	℥	ʒ	℈
2	11	6	0
10	8	3	1
14	10	2	2
0	6	5	0
7	5	4	1
36	6	5	1

ʒ	℈	*gr.*
1	0	18
2	1	15
3	2	13
4	0	0
6	1	7
18	0	13

AVOIRDUPOIS WEIGHT.

ton.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
10	18	2	25	15	1
1	15	0	0	14	15
12	0	1	3	0	10
0	13	0	27	1	11
2	2	2	0	7	8
27	9	3	1	7	13

cwt.	*qr.*	*lb.*	*oz.*
4	3	20	5
5	0	12	3
1	2	0	8
0	3	25	13
1	2	20	10
14	0	23	7

LONG MEASURE.

L.	*mi.*	*fur.*	*rd.*	*yd.*
1	2	6	37	4
6	0	0	30	5
0	1	4	0	3
2	0	1	1	0
3	2	0	25	1
14	0	5	15	2

rd.	*yd.*	*ft.*	*in.*
10	4	2	8
1	3	0	5
8	2	1	6
1	1	0	4
0	2	1	9
22	3	0	8

CLOTH MEASURE.

yd.	*qr.*	*na.*
15	1	2
13	0	3
20	2	2
0	3	0
8	1	1
58	1	0

E. Fl.	*qr.*	*na.*
3	2	3
15	1	2
9	2	0
8	0	1
10	0	0
47	0	2

E. E.	*qr.*	*na.*
4	2	2
10	1	1
9	2	0
13	0	2
15	1	1
52	2	2

SQUARE MEASURE.

Sq. yd.	*Sq. ft.*	*Sq. in.*
100	8	130
50	0	100
10	5	0
0	8	143
13	2	8
175	7	93

M.	*A.*	*R.*	*P.*
0	100	1	30
10	600	2	10
8	40	1	12
0	0	3	2
4	4	0	20
23	106	0	34

SOLID, OR CUBIC MEASURE.

S. yd.	*S. ft.*	*S. in.*
4	26	1000
1	10	1541
0	20	80
10	17	11
8	25	59
26	18	963

C.	*S. ft.*
10	120
8	100
2	80
0	119
12	6
35	41

C.	*Cord ft*
3	7
10	4
12	1
8	6
15	3
50	5

WINE MEASURE.

hhd.	*gal.*	*qt.*	*pt.*
4	30	3	1
10	25	0	1
25	0	2	0
0	60	0	1
13	45	3	0
54	36	1	1

tun.	*pi.*	*hhd.*	*gal.*	*qt.*	*pt.*	*gi.*
1	1	1	37	3	1	3
10	0	0	50	0	1	2
11	0	1	13	1	0	1
4	1	0	25	2	0	0
8	0	1	18	0	1	3
36	0	1	19	0	1	1

ALE, OR BEER MEASURE.

hhd.	*gal.*	*qt.*	*pt.*
2	50	3	1
10	30	1	0
11	25	0	1
25	1	1	0
6	52	3	1
56	52	1	1

bar.	*gal.*	*qt.*
10	30	1
6	20	0
1	5	2
10	0	3
4	35	1
33	19	3

DRY MEASURE.

ch.	*bu.*	*pk.*	*qt.*	*pt.*
1	30	3	7	1
0	35	2	3	0
10	19	1	0	1
5	10	2	4	0
4	4	0	5	1
22	28	2	4	1

bu.	*pk.*	*qt.*	*pt.*
10	1	1	1
2	3	6	0
5	2	3	0
8	0	0	1
15	2	4	0
42	1	7	0

TIME.

da.	*hr.*	*m.*	*sec.*
15	18	50	49
1	13	59	59
4	23	0	2
10	11	1	4
0	2	10	15
32	21	2	9

wk.	*da.*	*hr.*	*m.*	*sec.*
1	2	13	40	30
2	6	10	8	3
0	5	22	55	45
2	3	4	1	15
1	2	4	5	0
8	6	6	50	33

CIRULAR MEASURE, OR MOTION.

cr.	s.	°	′	″
1	8	25	40	35
0	11	1	2	43
1	0	29	59	0
0	1	10	13	5
0	2	5	4	3
4	0	11	59	26

s.	°	′
1	25	2
0	18	50
2	5	39
0	4	4
4	15	10
9	8	45

°	′	″
13	10	19
1	40	35
2	48	39
0	30	40
10	45	45
28	55	58

SUBTRACTION OF DENOMINATE NUMBERS.

83. If we wish to subtract £15 13*s.* 10*d.* from £20 5*s.* 8*d.*, we proceed as follows:

OPERATION.

£	*s.*	*d.*
20	5	8
15	13	10
4	11	10

We place the numbers of the subtrahend directly under the numbers of the same denomination in the minuend, and draw a line underneath. Commencing with the pence, we see that we cannot subtract 10*d.* from 8*d.*; we therefore increase the 8*d.* by 12*d.* making 20*d.*; then subtracting 10*d.* from the 20*d.*, we have the difference 10*d.*, which we write under the *column* of pence. Having added 12*d.* to the minuend, we *must equally* increase the subtrahend, which we do by adding 1*s*. (the same as the 12*d*.,) to the 13*s*., making 14*s*. This cannot be subtracted from 5*s*.; we therefore increase the 5*s*. by 20*s*., making 25*s*. Now, subtracting 14*s*. from 25*s*. we have 11*s*., which we write under the column of shillings. Before subtracting the pounds, we add £1 to £15 to compensate for the 20*s*. added to the 5*s*., and then say £16 from £20 leaves £4.

NOTE.—It will be seen that this process is similar to that in the "shorter and more practical" example of simple subtraction, (ART. **12**.) But the preceding subtraction might be also performed as in the second example of simple subtraction.

Hence, we have this general

RULE.

I. Place the less number under the greater, so that the same denominations may stand under each other; draw a line below them.

II. Begin at the right, and subtract each number in the lower line from the one directly above it, and set the remainder below.

III. If any number in the lower line is greater than the one above it, add so many to the upper number as make one of the next higher denomination; then subtract the lower number from the upper one thus increased, and set down the remainder. Carry 1, *expressing the increase of the upper line, to the next number in the lower line; after which subtract this number from the one above it, as before; and thus proceed till all the numbers are subtracted.*

PROOF.

If the work be right, the difference added to the subtrahend will equal the minuend; as in simple subtraction.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
13	18	1	20	0	13
10	0	3	21	12	0
3	17	1	26	4	13

A.	*R.*	*P.*
69	3	25
10	0	38
59	2	27

℔	℥	ʒ	℈	*gr.*
24	7	2	1	16
16	10	3	2	17
7	8	6	1	19

L.	*mi.*	*fur.*	*rd.*
16	2	7	39
5	0	7	8
11	2	0	31

E.	*Fr.*	*qr.*	*na.*
10	5	0	
5	1	3	
5	3	1	

ch.	*bu.*	*pk.*	*qt.*	*pt.*
30	10	1	1	0
10	8	3	6	1
20	1	1	2	1

tun.	*pi.*	*hhd.*	*gal.*	*qt.*
10	1	1	50	1
1	0	0	60	3
9	1	0	52	2

da.	*hr.*	*m.*	*sec.*
100	10	0	1
60	0	40	45
40	9	19	16

yr.	*mo.*	*wk.*	*da.*
17	8	3	1
4	1	2	6
13	7	0	2

mi.	*fur.*	*rd.*
60	0	0
40	7	39
19	0	1

C.	*S. ft.*
45	126
10	127
34	127

C.	*Cord ft.*
100	6
80	7
19	7

£	*s.*	*d*
50	0	1
30	10	10
19	9	3

84. EXERCISES IN ADDITION AND SUBTRACTION.

1. Bought 20 yards of broadcloth for £18 5*s.* 3*d.*, 30 *pounds of* feathers for £8 2*s.* 4d., 100 yards carpeting for

£45 17*s*. 8*d*., 10 pieces of cotton cloth for £8 18*s*. 1*d*., 50 yards of calico for £2 0*s*. 10*d*. What was the cost of the whole? *Ans*. £83 4*s*. 2*d*.

2. Bought four hogsheads of sugar, weighing as follows: 1st weighed 8*cwt*. 1*qr*. 23*lb*. 10*oz*.; 2d weighed 9*cwt*. 2*qr*. 0*lb*. 3*oz*.; 3d weighed 10*cwt*. 0*qr*. 0*lb*. 8*oz*.; 4th weighed 8*cwt*. 3*qr*. 27*lb*. How much did the four weigh? *Ans*. 36*cwt*. 3*qr*. 23*lb*. 5*oz*.

3. A man owns three farms; the first contains 69 acres, 3 roods, 10 rods; the second contains 100 acres, 5 rods; the third contains 150 acres, 2 roods. How many acres are there in all? *Ans*. 320*A*. 1*R*. 15*P*.

4. Suppose a note given August 3d, 1838, to be paid November 10th, 1843. How long was the note on interest, if we count 30 days to the month? and how long if the time is accurately computed? 1st *Ans*. 5*yr*. 3*mo*. 7*da*.
2d *Ans*. 1925 days.

5. A person buys 15*cwt*. 3*qr*. 20*lb*. of sugar, and sells 10*cwt*. 0*qr*. 11*lb*. How much remains unsold? *Ans*. 5*cwt*. 3*qr*. 9*lb*.

6. From a piece of cloth containing 37*yd*. 3*qr*. 2*n*., there has been taken at one time, 6*yd*. 1*qr*., at another time 10*yd*. 3*qr*. 3*na*. How much then remains? *Ans*. 20*yd*. 2*qr*. 3*na*.

7. From a pile of wood containing 100 cords, I sold at one time 10*C*. 100*S.ft*., at another time I sold 18*C*. 59*S.ft*. How many cords remain unsold? *Ans*. 70*C*. 97*S.ft*.

8. A farmer raises 100*bu*. 3*pk*. 2*qt*. of wheat from one field, 87*bu*. 1*pk*. 1*qt*. 1*pt*. from another field; he sells 53*bu*. to one person, and 37*bu*. 2*pk*. 1*qt*. to another person. How many bushels has he remaining? *Ans*. 97*bu*. 2*pk*. 2*qt*. 1*pt*.

9. Bought 5 loads of coal. The first weighed 2056 pounds, the second weighed 2250, the third weighed 2240,

the fourth weighed 2310, the fifth weighed 2330. What was the entire weight? And how many tons of 2000 pounds each?

Ans. { 11186 pounds.
5·593 tons.

10. A person engages to build 100 rods, and 10 feet of stone fence. At one time he builds 17 rods, 5 feet; at another time he builds 37 rods, 15 feet. How much still remains to be built? *Ans.* 45 rods, $6\frac{1}{2}$ feet.

11. How much cloth in three pieces, measuring as follows: first piece 37 yards, 3 quarters, 1 nail; second piece 41 yards, $1\frac{1}{2}$ Flemish Ells; third piece 43 yards, $1\frac{1}{2}$ English Ells? *Ans.* 124*yds.* 3*qr.* 1*na.*

12. Bought 3 loads of wood; the first was 8 feet long, 4 feet wide, and 3 feet high; the second was 7 feet long, 4 feet wide, and 2 feet high; the third was 9 feet long, 3 feet wide, and 3 feet high. How many solid feet in the whole? How many cord feet, and how many cords?

Ans. { 233 cubic feet.
14 cord feet, 9 cubic feet.
1 cord, 6 cord feet, 9 cubic feet.

MULTIPLICATION OF DENOMINATE NUMBERS.

85. If we wish to multiply £13 5*s.* 10*d.* by 5, we proceed as follows:

OPERATION.

£	*s.*	*d.*
13	5	10
		5
66	*9*	*2*

First, we say 5 times 10*d.* is 50*d.*, which equals 4*s.* and 2*d.*; we set down the 2*d.* and reserve the 4*s.* to carry into the next column. We then say 5 times 5*s.* equals 25*s.*, to which adding the 4*s.* we have 29*s.*, which equals £1 9*s.*; we set down the 9*s.* and reserve the £1 to carry to the next denomination. Finally, we say 5 times £13 is £65, to which adding the £1, we have £66; this being the highest denomination, we set it down entire.

Hence this general

RULE.

I. Set the multiplier under the lowest denomination of the multiplicand, and draw a line below it.

II. Multiply the number in the lowest denomination by the multiplier; divide the product by the number expressing how many it takes of such denomination to make one of the next higher. Write the remainder under the number multiplied, and reserve the quotient. Then multiply the number in the next higher denomination by the multiplier, and to the product add the reserved quotient. Divide as before, writing down the remainder, and carrying the quotient.

III. Proceed in like manner to the highest denomination, of which the entire product must be set down.

In Multiplication of Denominate Numbers, where do you set the multiplier? Which denominate value do you first multiply? After finding in the product the number of units of next higher order and also what remains, where do you place the remainder? and what do you do with the units of next superior order? Repeat the rest of the Rule.

EXAMPLES.

(1.)

£	*s.*	*d.*
10	10	10
		3
31	*12*	6

(2.)

	cwt.	*qr.*	*lb.*	*oz.*	*dr.*
	8	0	2	4	5
T.					6
2	8	0	13	9	14

3. In 3 hogsheads of sugar, each containing 10*cwt.* r. 5*lb.*, how many hundred weight?

Ans. 32*cwt.* 1*qr.* 15*lb.*

4. How much cloth will it take for 7 suits of clothes, each suit require 7*yd.* 3*qr.* 1*na.*?

Ans. 54*yd.* 2*qr.* 3*na.*

5. How much wood can a horse draw in 13 loads, if he w 1*C.* 19*S.ft.* at each load? *Ans.* 14*C.* 119*S.ft.*

5. How long will it take a man to saw 6 cords of wood, he employ 7*hr.* 30*m.* 45*sec.* to saw one cord, allowing working hours for each day?

Ans. 4*da.* 5*hr.* 4*m.* 30*sec.*

7. The circumference of a wheel is 15 feet 2 inches. hat distance will this wheel measure on the ground, if s rolled over 365 times? *Ans.* 1*mi.* 255*ft.* 10*in.*

3. Allowing the year to consist accurately of 365 days, hours, 48 minutes, 49½ seconds, what will be the true gth of 1843 years? *Ans.* 673141*da.* 10*hr.* 44*m.* 28½*sec.*

When the multiplier is a composite number, we may, in simple numbers, multiply successively by the com-ient parts.

). What will 35*cwt.* of cheese cost, at 15*s.* 6*d.* per ndred weight?

OPERATION.

£	s.	d.	
	15	6	cost of 1*cwt.*
		5	
3	17	6	cost of 5*cwt.*
		7	×7
27	2	6	cost of 35*cwt.*

10. How much brandy in 84*pi.*, each containing 128*gal* 2*qt.* 1*pt.* 3*gi.*? *Ans.* 10812*gal.* 1*qt.* 1*pt.*

11. In 21 loads of wood, each 1*C.* 1*C.ft.*, how many cords? *Ans.* 23*C.* 5*C.ft.*

12. Suppose the piston rod of a steam engine to move 3*ft.* 4*in.* 1*b. c.* at each stroke. Through what distance will it move in making 1000 strokes? *Ans.* 3361*ft.* 1*in.* 1*b. c.*

13. Bought as follows:

lb.			*s.*	*d.*	
18	of green tea,	at	12	3	per pound.
12	of raisins,	"	1	2	" "
27	of loaf sugar,	"	1	4	" "
15	of English currants,	"	2	3	" "
14	of citron,	"	3	6	" "

What is the amount of the whole purchase?

Ans. £17 13*s.* 3*d.*

14. What is the amount of the following bill of goods?

			£	*s.*	*d.*	
15	yards of broadcloth,	at	1	3	6	per yard.
12	" " silk,	"		18	3	" "
20	" " calico,	"		1	9	" "
24	" " sheeting,	"		1	3	" "
22	" " muslin,	"		3	4	" "

Ans. £29 9*s.* 10*d.*

DIVISION OF DENOMINATE NUMBERS.

86. Let it be required to divide £100 10*s.* 3*d.* equally among 17 men.

EXPLANATION.

First, we say 17 in £100, is contained 5 times and £15 remaining; and since these £15, as well as the 10*s*., are yet to be divided among the 17 men, we reduce the pounds to shillings, and add the 10*s*., making 310*s*.; we find 17 to be contained 18 times in 310*s*. with 4*s*. remainder. We reduce the 4*s*. to pence, and add the 3*d*., making 51*d*., which divided among the 17 men, gives 3*d*. each.

OPERATION.

17)£100 10*s*. 3*d*.(£5
85
15
20
17)310(18*s*.
17
140
136
4
12
17)51(3*d*.
51

Collecting, we have £5 18*s*. 3*d*.

NOTE.—We do not divide 100 pounds by 17 men, which is impossible; but we separate £100 into 17 equal parts. Each part is expressed by the quotient, and contains £5, (ART. 64. Note.)

Or, adhering to the general definition of Division, (ART. 22 and ART. 64,) we suppose a pound set apart for each man, and then find how many times £17, the number thus set apart, is contained in £100; the quotient will be an abstract number. The answer will, of course, be as many pounds to each man as there are parcels of £17 in £100; that is, as there are units in the quotient.

Had the divisor been one of the nine digits, the work might have been performed by Short Division.

We therefore have this general

RULE.

I. Place the divisor on the left of the dividend, as in Simple Division. Begin at the left-hand and divide the number of the highest denomination by the divisor. Reduce the re-

inder, if any, to the next lower denomination, to which add number of the dividend expressing that denomination, d then divide the sum by the divisor.

II. Proceed in the same way for all the denominations. If re is a remainder after the last division, place it over the isor, and annex it in a fractional form to the quotient. Each tient will be of the same denomination as its dividend.

aving placed the divisor as in Simple Division, how do you proceed? When, in ling any particular denomination, there is a remainder, how do you dispose of it? hat denomination will the respective quotients be?

EXAMPLES.

(1.)

$$\begin{array}{rrrr} & \text{yd.} & \text{qr.} & \text{na.} \\ 7) & 25 & 3 & 1 \\ \hline & 3 & 2 & 3 \end{array}$$

(2.)

$$\begin{array}{rrrrrr} & \text{cwt.} & \text{qr.} & \text{lb.} & \text{oz.} & \text{dr.} \\ 9) & 27 & 3 & 26 & 13 & 9 \\ \hline & 3 & 0 & 12 & 5 & 1 \end{array}$$

(3.)

$$\begin{array}{rllll} & \text{lb.} & \text{oz.} & \text{pwt.} & \text{gr.} \\ 13) & 10 & 8 & 16 & 3(0\text{lb.}\,9\text{oz.}\,18\text{pwt.}\,3\tfrac{12}{13}\text{gr.} \\ & 12 & & & \\ \hline 13) & 128(9\text{oz.} & & & \\ & 117 & & & \\ \hline & 11 & & & \\ & 20 & & & \\ \hline 13) & 236(18\text{pwt.} & & & \\ & 13 & & & \\ \hline & 106 & & & \\ & 104 & & & \\ \hline & 2 & & & \\ & 24 & & & \\ \hline 13) & 51(3\tfrac{12}{13}\text{gr.} & & & \\ & 39 & & & \\ \hline & 12 \text{ remainder.} & & & \end{array}$$

(4.)

```
       mi. fur. rd. yd. ft.mi.fur. rd. yd. ft   in.
   23)100   4   30  1½   2(4  2   39   3   0  7 7/23
       92
       --
        8
        8
     -------
   23)68(2fur.
       46
       --
       22
        40
     --------
   23)910(39rd.
       69
       ---
       220
       207
       ---
        13
         5½
      ------
   23)73  (3yd.
       69
       --
        4
        3
      -----
   23)14(0ft.
       12
      -----
   23)168( 7 7/23 in.
      161
      ---
        7 remainder.
```

5. Divide 10*tuns* 2*hhd.* 17*gal.* 2*pt.* by 67.
Ans. 39*gal.* 6*pt.*

6. Divide 51*A.* 1*R.* 11*P.* by 51. *Ans.* 1*A.* 0*R* 1*P.*

7. Divide 4*gal.* 2*qt.* by 144. *Ans.* 1*gi.*

8. Divide £113 13*s.* 4*d.* by 31. *Ans.* £3 13*s.* 4*d.*

9. Divide 673141*da.* 9*hr.* 58*m.* 24*sec.* by 1843.
Ans. 365*da.* 5*h.* 48*m.* 48*sec.*

10. Divide 1*mi.* 255*ft.* 10*in.* by 365. *Ans.* 15*ft.* 2*in.*

When the divisor is a composite number, we may divide by the factors of the number successively.

11. Bought 15 sheep for £5 12*s.* 6. How much did one sheep cost?

FIRST OPERATION.	SECOND OPERATION.
£ *s.* *d.*	£ *s.* *d.*
3)5 12 6 cost of 15 sheep.	5)5 12 6 cost of 15 sheep.
5)1 17 6 cost of 5 sheep.	3)1 2 6 cost of 3 sheep.
0 7 6 cost of 1 sheep.	0 7 6 cost of 1 sheep.

From this example, we see that it makes no difference which factor is first used.

12. If 24*yds.* of cloth cost £18 6*s.*, how much is that per yard? *Ans.* 15*s.* 3*d.*

13. From a piece of cloth containing 128*yds.* 1*qr.*, a tailor made 18 coats, which took one third of the whole piece. How many yards did each coat contain?
Ans. 2*yds.* 1*qr.* 2*na.*

87. QUESTIONS INVOLVING THE FOUR PRECEDING RULES.

1. Twenty-four men agree to construct 7*mi.* 1*fur.* 24*rd.* of road; after completing $\frac{1}{6}$ of it, they employ 8 more men. What distance does each man construct before and after the 8 men were employed?
Ans. { 16*rd.* before.
1*fur.* 20*rd.* after.

2. A silversmith has seven tea-pots, each weighing 1*lb.* 3*oz.* 13*pwt.* 11*gr.* What is the whole weight?
Ans. 9lb. 1oz. 14pwt. 5gr.

3. A farmer has 1000 bushels of apples, which he puts into 350 barrels. How many does each barrel hold?

Ans. 2*bu.* 3*pk.* $3\frac{3}{7}$*qt.*

4. If it require 1 sheet of paper to print 24 pages of a book, how many reams, allowing 18 quires to the ream, will it take to print 3000 copies, of 250 pages each?

Ans. 72 reams, 6 quires, 2 sheets.

5. An estate worth £2570 is to be divided as follows: the widow has one third of the whole, the remainder is to be divided equally between seven children. How much does the widow receive, and how much does each child have?

Ans. The widow has £856 13*s.* 4*d.*
Each child has £244 15*s.* 2*d.* $3\frac{3}{7}$ *far.*

6. Divide 100 acres, 3 roods, 8 rods of land, between four persons, A, B, C, and D, so that A shall have one sixth of the whole, B one fourth of the remainder, C one third of what then remains, and D the rest. How much does each one have?

Ans.			
A had	16*A.*	3*R.*	8*P.*
B had	21	0	0.
C had	21	0	0.
D had	42	0	0.

7. A, B, C, and D, having 13*cwt.* 1*qr.* 4*lb.* of sugar, they agree to divide it as follows: A is to have one half of the whole, B is to have one third of the remainder, C is to have one fourth of what then remains, and D is to take what is left. What were their respective portions?

Ans.			
A had	6*cwt.*	2*qr.*	16*lb.*
B had	2	0	24.
C had	1	0	12.
D had	3	1	8.

8. What is the weight of the following coins: 10 guineas, each weighing, 5 pwt. $9\frac{1}{2}$ grains; 7 sovereigns, each weighing 1 pwt. $8\frac{1}{4}$ grains?

Ans. 3*oz.* 3*pwt.* $8\frac{3}{4}$*gr.* of gold.

9. What is the weight of 13 crowns, each weighing 18 pwt. $4\frac{4}{11}$ grains; 14 shillings, each weighing 3 pwt. $15\frac{2}{11}$ gr.; 9 sixpences, each weighing 1 pwt. $19\frac{7}{11}$ gr.?

Ans. 1*lb.* 3*oz.* 3*pwt.* $15\frac{3}{11}$*gr.* of silver.

10. In one eagle there is $232\frac{2}{10}$ grains of pure gold, $12\frac{9}{10}$ grains of silver, and $12\frac{9}{10}$ grains of copper, and the same proportions of gold, silver and copper, from all other American gold coin. In 10 eagles, 7 half-eagles, 5 quarter-eagles, how many grains of gold, silver and copper?

$$Ans. \begin{cases} 3424{\cdot}95 \ gr. \text{ of gold.} \\ 190{\cdot}275 \ gr. \text{ of silver.} \\ 190{\cdot}275 \ gr. \text{ of copper.} \end{cases}$$

11. One pound of pure gold is sufficient for how many dollars of gold coin, if it require 23·22 grains for one dollar? *Ans.* 248·062 dollars.

12. One pound of pure silver is sufficient for how many dollars of silver coin, if it require 371·25 grains for one dollar? *Ans.* 15·515 dollars.

DENOMINATE FRACTIONS.

88. Under Art. **64**, we defined a denominate number as one whose unit has reference to a particular thing. For a similar reason, a *denominate fraction* is a part of a unit having reference to a particular thing. Thus, $\frac{1}{2}$ of a yard is a denominate fraction, expressing a part of the *particular unit one yard*; $\frac{3}{4}$ of a pound is also a denominate fraction, expressing a part of the particular unit one pound.

We know (by Art. **80**,) that denominate numbers may be changed or reduced from one denomination to another without altering their values. By a similar method may denominate fractions be reduced from one denomination to another.

What have we already defined a denominate number to be? What is a denominate fraction? Give some examples. May denominate fractions be changed from one name to another without altering their values?

REDUCTION OF DENOMINATE FRACTIONS.

89. Suppose we wish to reduce $\frac{1}{360}$ of a pound sterling to an equivalent fraction of a farthing, we proceed as follows: since there are 20 shillings in a pound, $\frac{1}{360}$ of a pound is the same as 20 times $\frac{1}{360}$ of a shilling; and this is the same as 12 times 20 times $\frac{1}{360}$ of a penny; which, in turn, is 4 times 12 times 20 times $\frac{1}{360}$ of a farthing. That is, $\frac{1}{360}$ of a pound sterling $=\frac{1}{360}$ of $\frac{20}{1}$ of $\frac{12}{1}$ of $\frac{4}{1}$ of a farthing $=$, by calculation, to $\frac{8}{3}$ of a farthing.

Again, let us reduce $\frac{8}{5}$ of a farthing to a fraction of a pound sterling. In this case, we reverse the preceding process, and instead of multiplying, divide by the same fractions; or what is the same thing, take the reciprocals of the fractions, (Art. **47**,) and multiply.

Thus $\frac{8}{5}$ of a farthing $=\frac{8}{5}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound sterling $=\frac{1}{1600}$ of a pound sterling.

1. Reduce $\frac{3}{8}$ of an inch to the fraction of a mile.

The increase of denominate value between the inch and the mile, is for the foot 12 times the inch, for the rod *$16\frac{1}{2}$ or $\frac{33}{2}$ times* the foot, for the furlong 40 times the rod, and for the mile 8 times the furlong. Therefore a compound fraction representing what part of a mile an inch is, would be $\frac{1}{12}$ of $(1\div\frac{33}{2}=)\frac{2}{33}$ of $\frac{1}{40}$ of $\frac{1}{8}$. So that the fraction $\frac{3}{8}$ of an inch, which is to be changed to the fraction of a mile, must be multiplied by the compound fraction just obtained. Consequently we have

$$\frac{3}{8}\text{ of an inch}=\frac{\cancel{3}}{8}\times\frac{1}{\underset{2}{\cancel{12}}}\times\frac{\cancel{2}}{33}\times\frac{1}{40}\times\frac{1}{8}=\frac{1}{168960}\text{ of a mile.}$$

If the question had been the reduction of $\frac{3}{8}$ of a mile to the fraction of an inch, the fraction would have been

$$\frac{3}{8}\text{ of a mile}=\frac{3}{8}\times\frac{12}{1}\times\left(\frac{16\frac{1}{2}}{1}=\right)\frac{33}{2}\times\frac{40}{1}\times\frac{8}{1}=23760\text{ in.}$$

From what has been done we may deduce this

RULE.

I. When the given fraction is to be reduced to a higher denomination, multiply it by a compound fraction, whose terms are the reciprocals of the numbers that indicate the increase in value of a unit of the successive denominations included between the denomination of the given fraction and the one to which it is to be reduced.

II. When the given fraction is to be reduced to a lower denomination, multiply it by a compound fraction, whose terms have units for their denominators, and for numerators the numbers that indicate the decrease in value of a unit of the successive denominations included between the denomination of the given fraction and the one to which it is to be reduced.

EXAMPLES.

2. Reduce $\frac{3}{11520}$ of a day to the fraction of a second.

In this example, the decrease in value of a unit of the successive denominations between a solar day and a second, are 24 (hours,) 60 (minutes,) and 60 (seconds.) Hence the compound fraction will be $\frac{24}{1}$ of $\frac{60}{1}$ of $\frac{60}{1}$, which, multiplied by the given fraction becomes

$$\frac{3}{11520}\times\frac{24}{1}\times\frac{60}{1}\times\frac{60}{1}.$$

Cancelling, successively, 60 and 24, factors common to numerator and denominator, we have first

$$\frac{3}{\begin{array}{c}\cancel{11520}\\192\end{array}}\times\frac{24}{1}\times\frac{\cancel{60}}{1}\times\frac{60}{1};\ \text{then}\ \frac{3}{\begin{array}{c}\cancel{11520}\\\cancel{192}\\8\end{array}}\times\frac{\cancel{24}}{1}\times\frac{\cancel{60}}{1}\times\frac{60}{1}.$$

Finally, cancelling the factor 4, which is common to the numerator 60, and the denominator 8, we have

$$\frac{3}{\begin{array}{c}\cancel{11520}\\\cancel{192}\\\cancel{8}\\2\end{array}}\times\frac{\cancel{24}}{1}\times\frac{\cancel{60}}{1}\times\frac{\overset{15}{\cancel{60}}}{1}=\frac{45}{2}\ \text{of a second.}$$

We have been particular to give the complete work of cancelling in these examples, by writing down the whole work at the successive stages of operation. In practice the expression need not be written more than once. With a little practice the pupil will be able to strike out the common factors with accuracy and despatch.

Reduce $\frac{1}{4032}$ of a pipe of wine to an equivalent fraction of a gill.

In this example, the successive denominate values between a pipe and a gill are 2 hogsheads, 63 gallons, 4 quarts, 2 pints and 4 gills; therefore, our compound fraction is $\frac{2}{1}$ of $\frac{63}{1}$ of $\frac{4}{1}$ of $\frac{2}{1}$ of $\frac{4}{1}$, which, multiplied by the given fraction, becomes $\frac{1}{4032}$ of $\frac{2}{1}$ of $\frac{63}{1}$ of $\frac{4}{1}$ of $\frac{2}{1}$ of $\frac{4}{1}$; this becomes, after cancelling like factors, 1 gill.

4. Reduce $\frac{144}{360}$ of a yard to a fraction of a mile. *Ans.* $\frac{1}{4400}$.

5. Reduce $\frac{27}{84}$ of a gill to the fraction of a gallon. *Ans.* $\frac{9}{896}$.

6. Reduce $\frac{330}{495}$ of a pound to the fraction of a ton. *Ans.* $\frac{1}{3360}$.

7. Reduce $\frac{1}{3}$ of a mile to the fraction of a foot. *Ans.* 1760 feet.

8. Reduce $\frac{1}{3}$ of $\frac{1}{3}$ of $\frac{3}{7}$ of a yard to the fraction of a mile. *Ans.* $\frac{1}{36960}$.

9. Reduce $\frac{1}{6}$ of $\frac{1}{8}$ of $\frac{24}{28}$ of a gallon to the fraction of a gill. *Ans.* $\frac{4}{7}$.

10. Reduce $\frac{2}{3}$ of $\frac{4}{9}$ of a hogshead of wine to the fraction of a gill. *Ans.* $\frac{1792}{3}=597\frac{1}{3}$.

11. Reduce $\frac{1}{2}$ of $\frac{3}{7}$ of $4\frac{1}{2}$ yards to the fraction of an inch. *Ans.* $\frac{243}{7}=34\frac{5}{7}$.

12. Reduce $\frac{1}{7}$ of $\frac{3}{43}$ of a farthing to the fraction of a shilling. *Ans.* $\frac{1}{4816}$.

13. Reduce $\frac{7}{59}$ of an ounce to the fraction of a pound avoirdupois. *Ans.* $\frac{7}{944}$.

14. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of 1 rod to the fraction of an inch, of a foot, and of a yard.

$$\textit{Ans.}\begin{cases}\frac{2079}{16}=129\frac{15}{16}\text{ inches.}\\ \frac{693}{64}=10\frac{53}{64}\text{ feet.}\\ \frac{231}{64}=3\frac{39}{64}\text{ yards.}\end{cases}$$

15. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of 1 hour to the fraction of a month of 30 days, and to the fraction of a year of 365 days.

$$\textit{Ans.}\begin{cases}\frac{1}{2160}\text{ of a month.}\\ \frac{1}{26280}\text{ of a year.}\end{cases}$$

90. To find what fractional part one quantity is of another of the same kind, but of different denominations.

Suppose we wish to know what part of 1 yard, 2 feet 3 inches is. We reduce 1 yard to inches, which gives 1 yard$=$36 inches; we also reduce 2 feet 3 inches to inches, which gives 2 feet 3 inches$=$27 inches. Now, it is obvious that 2 feet 3 inches is the same part of a yard that 27 is of 36, which is $\frac{27}{36}=\frac{3}{4}$.

Hence, we deduce this

RULE.

Reduce the given quantities to the lowest denomination mentioned in either. Then take the number which expresses the quantity of which the other is to be the fractional part, for a denominator, and the other number for a numerator, and the fraction thus formed will denote the fractional part sought.

EXAMPLES.

1. What part of £3 4*s.* 1*d.* is 2*s.* 6*d.*?

In this example, the quantities, when reduced, become £3 4*s.* 1*d.*$=$769*d.*; and 2*s.* 6*d.*$=$30*d.*; therefore, $\frac{30}{769}$ is the fractional part which 2*s.* 6*d.* is of £3 4*s.* 1*d.*

2. What part of 3 miles, 40 rods, is 27 feet 9 inches? *Ans.* $\frac{37}{22000}$.

3 What part of a day is 17 minutes 4 seconds? *Ans.* $\frac{8}{675}$.

4. What part of $700 is $5·30? *Ans.* $\frac{53}{7000}$.

5. What fractional part of 2 hogsheads is 3 pints? *Ans.* $\frac{1}{336}$.

6. What part of $3 is 2½ cents? *Ans* $\frac{1}{120}$.

7. What part of 10 shillings, 8 pence, is 3 shillings penny? *Ans.* $\frac{37}{128}$.

8. What part of 100 acres is 63 acres, 2 roods, 7 rods of land? *Ans.* $\frac{10167}{16000}$.

9. In the Eagle there are 232$\frac{1}{5}$ grains of pure gold, and 12$\frac{9}{10}$ grains of silver, and the same quantity of copper. The silver and copper is each what part, by weight, of the gold? And the silver and copper together is what part of the gold?

Ans. Silver and copper are each $\frac{1}{18}$ of the gold. Silver and copper together are $\frac{1}{9}$ of the gold.

10. In the United States standard silver coin of one dollar, there are 371$\frac{1}{4}$ grains of pure silver, and 41$\frac{1}{4}$ grains of copper. What fractional part is the copper of the silver? *Ans.* $\frac{1}{9}$.

11. The silver in standard gold coin is what part of the silver in the same value of standard silver coin? *Ans.* $\frac{43}{12375}$.

12. The pound Troy contains 5760 grains, the pound Avoirdupois contains 7000 grains. A pound Troy is what part of a pound Avoirdupois? *Ans.* $\frac{144}{175}$.

13. The imperial gallon contains 277$\frac{1}{4}$ cubic inches, nearly; the old wine gallon contains 231. What part of the imperial gallon is the old wine gallon? *Ans.* $\frac{924}{1109}$.

14. The solar year is 365 days, 5 hours, 48 minutes, 48 seconds. By what part of a day does this exceed 365 days? *Ans.* $\frac{109}{450}$.

91. To reduce a fraction of any given denomination to whole denominate numbers.

Suppose we wish to know the value of $\frac{3}{8}$ of a yard; we know that $\frac{3}{8}$ of a yard equals $\frac{3}{8}$ of $\frac{4}{1}$ of a quarter $=\frac{12}{8}$ of a *quarter* $=$ *1 quarter* $+\frac{1}{2}$ of a quarter. The $\frac{1}{2}$ of a quarter *may be considered* as a remainder.

Again, $\frac{1}{2}$ of a quarter equals $\frac{1}{2}$ of $\frac{4}{1}$ of a nail $=2$ nails. Therefore, $\frac{3}{8}$ of a yard equals 1 quarter and 2 nails.

Hence, we deduce this

RULE.

Multiply the numerator by the number expressing how many of the next lower denomination make one of the denomination of the fraction, and divide the product by the denominator; multiply the remainder, if any, by the number expressing how many of the next lower denomination make one of that remainder, and again divide the product by the denominator; continue this process until there is no remainder, or until we reach the lowest denominate value. The successive quotients will form the whole denominate numbers required.

EXAMPLES.

1. What is the value of $\frac{3}{15}$ of an hour?

In this example, $\frac{3}{15}$ of an hour equal $\frac{3}{15}$ of $\frac{60}{1}$ of a minute, equals 12 minutes.

2. What is the value of $\frac{3}{7}$ of 1 yard?

Ans. 1 quarter, $2\frac{6}{7}$ nails.

3. What is the value of $\frac{1}{2}$ of $\frac{3}{8}$ of 1 mile?

Ans. 1 furlong, 20 rods.

4. What is the value of $\frac{3}{7}$ of $\frac{5}{6}$ of 1 cwt.?

Ans. 1 quarter, 12 pounds.

5. What is the value of $\frac{1}{6}$ of 14 miles, 6 furlongs?

Ans. 2 miles, 3 furlongs, 26 rods, 11 feet.

6. What is the value of $\frac{1}{3}$ of $\frac{3}{5}$ of 2 days of 24 hours each?

Ans. 9 hours, 36 minutes.

7. What is the value of $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{3}{14}$ of an hour?

Ans. 5 minutes, $37\frac{1}{2}$ seconds.

8 What is the value of $\frac{109}{450}$ of a solar day?

Ans. 5*h.* 48*m.* 48*sec.*

9. What is the value of $\frac{14}{17}$ of a pound Avoirdupois?

Ans. 13*oz.* 2$\frac{14}{17}$*dr.*

10. What is the value of $\frac{1}{10}$ of a bushel? *Ans.* 3$\frac{1}{5}$ quarts.

11. What is the value of $\frac{6}{73}$ of a year of 365 days?

Ans. 30 days.

12. What is the value of $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ of an acre?

Ans. 25 rods.

ADDITION OF DENOMINATE FRACTIONS.

92. So long as fractions are of different denominate values, they cannot be added, any more than integers can of different denominate values. Hence, before seeking to add, it is necessary to reduce them to the same denomination, then, to a common denominator, and apply the rule under ART. **43.**

What is the Rule for the Addition of Denominate Fractions?

EXAMPLES.

1. Add $\frac{1}{5}$ of a shilling to $\frac{1}{4}$ of a pound.

I. $\frac{1}{5}$ of a shilling equals $\frac{1}{5}$ of $\frac{1}{20}$ of a pound $=\frac{1}{100}$ of a pound, which added to $\frac{1}{4}$ of a pound $=\frac{25}{100}$ of a pound, gives $\frac{26}{100}=\frac{13}{50}$ of a pound for the sum.

II. $\frac{1}{4}$ of a pound $=\frac{1}{4}$ of $\frac{20}{1}$ of a shilling $=5$ shillings, which, added to $\frac{1}{5}$ of a shilling, gives $5\frac{1}{5}=\frac{26}{5}$ of a shilling *for the sum.*

If our work is right, these two results ought to be of the same value, that is, $\frac{13}{50}$ of a pound must equal $5\frac{1}{5}$ shillings.

We know that $\frac{13}{50}$ of a pound $=\frac{13}{50}$ of $\frac{20}{1}$ of a shilling $=\frac{26}{5}$ of a shilling.

2. Add $\frac{1}{3}$ of a yard, $\frac{5}{6}$ of a foot, and $\frac{3}{8}$ of a mile.

These fractions, before adding, might be reduced to fractions of a yard, or of a foot, or of a mile, or of any of the denominate values of Long Measure. But a better way would be to reduce each to its integral denominate value, by Rule under ART. **91.**

Thus: $\frac{1}{3}$ of a yard $=\frac{1}{3}$ of $\frac{3}{1}$ of a foot $=1$ foot.

$\frac{5}{6}$ of a foot $=\frac{5}{6}$ of $\frac{12}{1}$ of an inch $=10$ inches.

$\frac{3}{8}$ of a mile $=\frac{3}{8}$ of $\frac{8}{1}$ of a furlong $=3$ furlongs.

Therefore, the sum is 3 furlongs, 1 foot, 10 inches.

3. Add $\frac{1}{2}$ of a week, $\frac{1}{6}$ of a day, $\frac{1}{4}$ of an hour.

$\frac{1}{2}$ of a week $=\frac{1}{2}$ of $\frac{7}{1}$ of a day $=3\frac{1}{2}$ days $=3$ days $+\frac{1}{2}$ of $\frac{24}{1}$ of an hour $=3$ days, 12 hours.

$\frac{1}{6}$ of a day $=\frac{1}{6}$ of $\frac{24}{1}$ hour $=4$ hours.

$\frac{1}{4}$ of an hour $=\frac{1}{4}$ of $\frac{60}{1}$ of a minute $=15$ minutes.

Hence, the sum is 3 days, 16 hours, 15 minutes.

4. Add $\frac{1}{5}$ of a year, $\frac{2}{7}$ of a week, $\frac{1}{12}$ of a day, together.

Ans. 75*da.* 2*hr.*

5. What is the sum of $\frac{1}{2}$ of a *cwt.*, $\frac{1}{3}$ of a *qr.*, $\frac{1}{4}$ of a *lb.*?

Ans. 2*qr.* 9*lb.* 9*oz.* $5\frac{1}{3}$*dr.*

6. What is the sum of $\frac{1}{10}$ of a bushel, $\frac{1}{6}$ of a peck, $\frac{1}{2}$ of a quart?

Ans. $5\frac{1}{30}$*qt.*

7. What is the sum of $\frac{1}{10}$ of a yard, and $\frac{1}{3}$ of a foot?

Ans. $7\frac{3}{5}$ inches.

8. What is the sum of $\frac{2}{3}$ of a week, $\frac{1}{5}$ of a day, and $\frac{1}{3}$ of an hour?

Ans. 4*da.* 21*hr.* 8*m.*

9. What is the sum of $\frac{2}{3}$ of a bushel, $\frac{1}{3}$ of a peck, and $\frac{1}{8}$ *of a quart?*

Ans. 3pk. 0qt. $0\frac{1}{4}$pt.

SUBTRACTION OF DENOMINATE FRACTIONS.

93. As in Addition, the fractions must be first reduced to the same denomination; afterwards they must be brought to a common denominator, and then the work may be completed, by Rule under ART. **44.**

What is the Rule for the Subtraction of Denominate Fractions.

EXAMPLES.

1. From $\frac{1}{8}$ of a pound subtract $\frac{1}{5}$ of a shilling.

I. $\frac{1}{8}$ of a £ $=\frac{1}{8}$ of $\frac{20}{1}$ of a shilling $=\frac{5}{2}$ of a shilling.

Therefore, $\frac{5}{2}-\frac{1}{5}=\frac{25}{10}-\frac{2}{10}=\frac{23}{10}$. So that the difference is $\frac{23}{10}$ of a shilling $=2\frac{3}{10}$ of a shilling.

II. $\frac{1}{5}$ of a shilling $=\frac{1}{5}$ of $\frac{1}{20}$ of a pound $=\frac{1}{100}$ of a pound.

And $\frac{1}{8}-\frac{1}{100}=\frac{25}{200}-\frac{2}{200}=\frac{23}{200}$. So that the difference is $\frac{23}{200}$ of a pound $=\frac{23}{200}$ of $\frac{20}{1}$ of a shilling $=\frac{23}{10}$ of a shilling, as before.

2. From $\frac{3}{8}$ of a day subtract $\frac{1}{5}$ of a minute.

$\frac{3}{8}$ of a day $=\frac{3}{8}$ of $\frac{24}{1}$ of an hour $=9$ hours.

$\frac{1}{5}$ of a minute $=\frac{1}{5}$ of $\frac{60}{1}$ of a second $=12$ seconds.

Hence, From 9*hr.* 0*m.* 0*sec.*
Take 0 0 12
Difference, 8 59 48

3. From $\frac{1}{2}$ of $\frac{2}{3}$ of 15 yards of cloth, subtract $\frac{1}{8}$ of $\frac{8}{15}$ of one quarter.

$\frac{1}{2}$ of $\frac{2}{3}$ of 15 yards $=5$ yards.

$\frac{1}{8}$ of $\frac{8}{15}$ of one quarter $=\frac{1}{8}$ of $\frac{8}{15}$ of $\frac{4}{1}$ of a nail $=\frac{4}{15}$ of a nail.

	yd.	*qr.*	*na.*
Hence, From	5	0	0
Take	0	0	$0\frac{4}{15}$
Difference,	4	3	$3\frac{11}{15}$

4. From $\frac{1}{7}$ of 5 acres of land, subtract $\frac{1}{4}$ of 3 roods. *Ans.* 2*R.* $4\frac{2}{7}$*P.*

5. From $\frac{2}{5}$ of an ounce, take $\frac{3}{8}$ of a pennyweight. *Ans.* 7*pwt.* 15*gr.*

6. From $\frac{1}{9}$ of a hogshead, take $\frac{2}{3}$ of a quart. *Ans.* 6*gal.* 3*qt.* $\frac{2}{3}$*pt.*

94. EXERCISES IN DENOMINATE FRACTIONS.

1. A person gave $\frac{1}{7}$ of a pound for a hat, $\frac{1}{3}$ of a shilling for some thread, and $\frac{1}{2}$ of a penny for a needle. What did he pay for all? *Ans.* 3*s.* 2*d.* $3\frac{1}{7}$*far.*

2. What is the value of $\frac{1}{2}$ of a week, $\frac{1}{3}$ of a day, and $\frac{1}{4}$ of a minute? *Ans.* 3*da.* 20*hr.* 15*sec.*

3. What is the value of $\frac{1}{5}$ of a pound, $\frac{1}{4}$ of an ounce, and $\frac{1}{7}$ of a pennyweight, Troy? *Ans.* 2*oz.* 13*pwt.* $3\frac{3}{7}$*gr.*

4. If $4\frac{1}{3}$ pounds of sugar cost $43\frac{1}{3}$ cents, how much is it per pound? *Ans.* 10 cents.

5. If I pay $4·04 for $8\frac{3}{4}$ bushels of apples, how much do I give per bushel? *Ans.* $46\frac{6}{35}$ cents.

6. Four persons, A, B, C, and D, own a ship, of which A owns $\frac{1}{5}$ of $\frac{2}{3}$ of the whole; B owns $\frac{7}{8}$ of $\frac{2}{3}$ as much as A; C owns $\frac{3}{4}$ as much as B; and D owns the remainder. What are the respective parts owned by each?

$$\textit{Ans.}\begin{cases}\text{A owned } \frac{48}{360}.\\ \text{B} \quad \text{"} \quad \frac{28}{360}.\\ \text{C} \quad \text{"} \quad \frac{21}{360}.\\ \text{D} \quad \text{"} \quad \frac{263}{360}.\end{cases}$$

7. From $\frac{1}{2}$ of $\frac{3}{4}$ of a day of 24 hours, take of $\frac{1}{3}$ of $1\frac{1}{2}$ hour. *Ans.* 8*h.* 30*m.*

8. To $\frac{3}{4}$ of $4\frac{1}{2}$ days of 24 hours each, add $\frac{1}{3}$ of $\frac{1}{6}$ of $3\frac{1}{2}$ hours. *Ans.* 3*d.* 9*d.* 11*m.* 40*sec.*

9. A certain sum of money is to be divided between 4 persons, in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is $28. What is the sum?

$\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{3}{4}$, which wants just $\frac{1}{4}$ of being the whole; hence, the fourth one had $\frac{1}{4}$ of the whole. Consequently, $28 is $\frac{1}{4}$ of the whole, and the whole is $\$28 \times 4=\$112$.

10. A received $\frac{1}{6}$ of a legacy, B $\frac{1}{10}$, and C the remainder. Now it is found that A had $80 more than B. How much did each receive?

$\frac{1}{6}-\frac{1}{10}=\frac{1}{15}$. Hence, $80 was $\frac{1}{15}$ of the whole legacy; the legacy was therefore $\$80 \times 15=\$1200$.

Hence, A had $\frac{1}{6}$ of $\$1200=\$200$.
B had $\frac{1}{10}$ of $\$1200=\$120$.
C had the remainder $=880$.

Proof, $1200.

11. Eight detachments of artillery divided 4608 cannon balls in the following manner: The first took 72 and $\frac{1}{9}$ of the remainder; the second took 144 and $\frac{1}{9}$ of the remainder; the third took 216 and $\frac{1}{9}$ of the remainder; the fourth took 288 and $\frac{1}{9}$ of the remainder. The balance was equally divided among the remaining four detachments. How many balls did each detachment receive?

Ans. Each received 576 balls.

12. Five persons divide 100 pounds of sugar as follows. The first takes $\frac{1}{7}$ of $\frac{3}{4}$ of the whole; the second takes $\frac{1}{6}$ of $\frac{3}{4}$ of the remainder; the third takes $\frac{1}{5}$ of $\frac{3}{4}$ of the remainder; the fourth takes $\frac{1}{4}$ of $\frac{3}{4}$ of the remainder;

and the fifth had what was left. How much did each receive?

lb. *lb.* *lb.* *oz.* *dr.*

Ans. The 1st had $\frac{1536}{14336}$ of 100 $=\frac{3}{28}$ of 100 $=$ 10 11 $6\frac{6}{7}$.
" 2d had $\frac{1600}{14336}$ of 100 $=\frac{25}{224}$ of 100 $=$ 11 2 $9\frac{1}{7}$.
" 3d had $\frac{1680}{14336}$ of 100 $=\frac{15}{128}$ of 100 $=$ 11 11 8.
" 4th had $\frac{1785}{14336}$ of 100 $=\frac{255}{2048}$ of 100 $=$ 12 7 $3\frac{1}{2}$.
" 5th had $\frac{7735}{14336}$ of 100 $=\frac{1105}{2048}$ of 100 $=$ 53 15 $4\frac{1}{2}$.

VULGAR FRACTIONS REDUCED TO DECIMALS.

95. To change a vulgar fraction into an equivalent decimal fraction.

Let us endeavor to change $\frac{3}{8}$ into an equivalent decimal fraction.

This fraction is the same as $\frac{3}{8}$ of a unit; and as 10 tenths make a unit, the fraction is the same as $\frac{3}{8}$ of $\frac{10}{1}$ of a tenth, $=3$ tenths $+\frac{6}{8}$ of a tenth. Again, $\frac{6}{8}$ of a tenth is the same as $\frac{6}{8}$ of $\frac{10}{1}$ of one hundredth, $=7$ hundredths $+\frac{4}{8}$ of one hundredth. But $\frac{4}{8}$ of one hundredth is the same as $\frac{4}{8}$ of $\frac{10}{1}$ of one thousandth, $=5$ thousandths. Therefore $\frac{3}{8}$ of a unit $=3$ tenths, 7 hundredths, and 5 thousandths, or as usually written, $\frac{3}{8}=0{\cdot}375$.

Hence we deduce this

RULE.

Annex a cipher to the numerator, and then divide by the denominator. If the dividend will not contain the divisor, write 0 in the quotient and annex another cipher, and then

divide; to the remainder annex another cipher, and again divide by the denominator; and so continue to do until there is no remainder, or until as many decimal figures have been obtained as may be desired. The quotient will be the decimal fraction required.

Note.—It will be seen that this rule bears a close analogy to rule under Art. 91, as it ought; since the values of the successive figures in a decimal fraction decrease in a tenfold ratio.

EXAMPLES.

1. What decimal fraction is equivalent to $\frac{1}{16}$?

$$\begin{array}{r} 16)100(0{\cdot}0625 \\ \underline{96} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ \underline{0} \end{array}$$

2. What decimal is equivalent to $\frac{1}{18}$?
Ans. 0·05555, &c.

3. What decimal is equivalent to $\frac{1}{20}$? *Ans.* 0·05.

4. What decimal is equivalent to $\frac{1}{25}$? *Ans.* 0·04.

5. What decimal is equivalent to $\frac{1}{3}$?
Ans. 0·3333, &c.

6. What decimal is equivalent to $\frac{1}{7}$?
Ans. 0·142857, &c.

7. What decimal is equivalent to $\frac{1}{11}$?
Ans. 0·0909, &c.

8. What decimal is equivalent to $\frac{1}{13}$?
Ans. 0·076923, &c.

9. What decimal is equivalent to $\frac{1}{17}$?

Ans. 0·0588235, &c.

10. Change $\frac{3}{4}$ into an equivalent decimal. *Ans.* 0·75.

11. Change $\frac{2}{3}$ into an equivalent decimal.

Ans. 0·6666, &c.

12. Change $\frac{3}{5}$ into an equivalent decimal. *Ans.* 0·6.

13. Change $\frac{5}{6}$ into an equivalent decimal.

Ans. 0·8333, &c.

14. Change $\frac{4}{7}$ into an equivalent decimal.

Ans. 0·5714285, &c.

15. Change $\frac{9}{16}$ into an equivalent decimal.

Ans. 0·5625.

16. Change $\frac{7}{8}$ into an equivalent decimal. *Ans.* 0·875.

17. Change $\frac{19}{20}$ into an equivalent decimal. *Ans.* 0·95.

18. Change $\frac{49}{50}$ into an equivalent decimal. *Ans.* 0·98.

19. Change $\frac{13}{14}$ into an equivalent decimal.

Ans. 0·928571428, &c.

In the foregoing process of converting a vulgar fraction into an equivalent decimal fraction, we continue to annex ciphers to the remainders, and to divide by the denominator of the vulgar fraction; hence, whenever we obtain a remainder like one that has previously occurred, then the decimal figures will commence a repetition. And as no remainder can exceed or equal the divisor or denominator of the vulgar fraction, the whole number of different remainders cannot *exceed* the number of units in the denominator less one; consequently, when the decimal figures do not terminate, they must recur in periods whose number of places cannot *exceed* the number of units less one in the denominator of the equivalent vulgar fraction.

Decimals which recur in this way, are called *repetends.* *When the* period begins with the first decimal figure, [illegible] is called a *simple repetend.* But when other decimal figures occur before the period commences, it is called a *compound repetend.*

A repetend is distinguished from ordinary decimals by a period or dot placed over the first and last figure of the circulating period.

96. The following vulgar fractions give simple repetends:

$\frac{1}{3}=0{\cdot}\dot{3}.$
$\frac{1}{7}=0{\cdot}\dot{1}4285\dot{7}.$
$\frac{1}{9}=0{\cdot}\dot{1}.$
$\frac{1}{11}=0{\cdot}\dot{0}\dot{9}.$
$\frac{1}{13}=0{\cdot}\dot{0}7692\dot{3}.$
$\frac{1}{17}=0{\cdot}\dot{0}58823529411764\dot{7}.$
$\frac{1}{19}=0{\cdot}\dot{0}5263157894736842\dot{1}.$
$\frac{1}{21}=0{\cdot}\dot{0}4761\dot{9}.$
$\frac{1}{23}=0{\cdot}\dot{0}43478260869565217391\dot{3}.$

97. The following ones give compound repetends:

$\frac{1}{6}=0{\cdot}1\dot{6}.$
$\frac{1}{12}=0{\cdot}08\dot{3}.$
$\frac{1}{14}=0{\cdot}0\dot{7}1428\dot{5}.$
$\frac{1}{15}=0{\cdot}0\dot{6}.$
$\frac{1}{18}=0{\cdot}0\dot{5}.$
$\frac{1}{22}=0{\cdot}0\dot{4}\dot{5}.$
$\frac{1}{24}=0{\cdot}041\dot{6}.$

98. Those simple repetends, which have as many terms, less one, as there are units in the denominator, we shall call *perfect repetends.* The following are some of the perfect repetends:

$\frac{1}{7}=0{\cdot}\dot{1}4285\dot{7}.$

$\frac{1}{17}=0{\cdot}\dot{0}58823529411764\dot{7}.$

$\frac{1}{19}=0{\cdot}\dot{0}5263157894736842\dot{1}.$

$\frac{1}{23}=0{\cdot}\dot{0}43478260869565217391\dot{3}.$

$\frac{1}{29}=0{\cdot}\dot{0}34482758620689655172413793\dot{1}.$

Note.—For some interesting properties of *repetends*, see Higher Arithmetic.

REDUCTION OF DENOMINATE DECIMALS.

99. A *denominate decimal* is a decimal fraction of a unit of a particular kind. Thus, 0·45 of a £, is a denominate decimal, since the unit is £1; for the same reason, 0·25 of a foot is a denominate decimal, the unit being 1 foot.

What is a denominate decimal? Give some examples.

CASE I.

To reduce denominate numbers of different denominations to a decimal of a given denomination.

Let it be required to reduce 15*s*. 6*d*. 3*far*. to the decimal of a £.

I. 3*far*.$=\frac{3}{4}d.=0{\cdot}75d.$

II. 6*d*. 3*far*. is therefore the same as 6·75*d*.; if we divide this by 12, it will become

$$\frac{6{\cdot}75}{12}=0{\cdot}5625s.$$

III. 15*s.* 6*d.* 3*far.*=15·5625*s.*; this divided by 20, gives

$$\frac{15{\cdot}5625}{20}=0{\cdot}778125 \text{ of a \pounds}$$

for the decimal sought. The work may be more concisely done, as in the following

OPERATION.

$$\begin{array}{r|l} 4 & 3\,far. \\ \hline 12 & 6{\cdot}75d. \\ \hline 2|0 & 15{\cdot}5625s. \\ \hline & 0{\cdot}778125 \text{ of a \pounds} \end{array}$$

EXPLANATION

We placed the different denominations above each other, so that the smallest denomination stood at the top; we then supposed ciphers annexed to the 3 farthings, and divided by 4, since 4 farthings make one penny, and the quotient, which must be a decimal, we placed at the right of the 6*d.*; we next divided 6·75*d.* with ciphers annexed, by 12, because 12 pence make one shilling, and the quotient, which is also a decimal, we placed at the right of the 15*s.*; finally, we divided the 15·5625*s.* by 20, because 20 shillings make one pound. In dividing by 20, we cut off the cipher, and then divided by 2, observing to remove the decimal point one place to the left.

We therefore have this

RULE.

Place the different denominations above each other, so that the lowest denomination may stand at the top; commencing at

the top, annex ciphers, and divide each denomination by the number expressing how many of such denomination make a unit of the next higher denomination. The last quotient will be the decimal required.

Repeat this Rule.

EXAMPLES.

1. Reduce £8 5*s*. 2*d*. 1*qr*. to the decimal of a £

OPERATION.

$$\begin{array}{r|l} 4 & 1 \\ \cline{2-2} 12 & 2{\cdot}25 \\ \cline{2-2} 2{\mid}0 & 5{\cdot}1875 \\ \cline{2-2} & \underline{8{\cdot}259375} \text{ of a £.} \end{array}$$

2. Reduce 3*qr*. 2*na*. to the decimal of a yard.

OPERATION.

$$\begin{array}{r|l} 4 & 2 \\ \cline{2-2} 4 & 3{\cdot}5 \\ \cline{2-2} & \underline{0{\cdot}875} \text{ of a yard.} \end{array}$$

3. Reduce 1*ft*. 4*in*. to the decimal of a yard.

OPERATION.

$$\begin{array}{r|l} 12 & 4 \\ \cline{2-2} 3 & 1{\cdot}3333, \text{ \&c} \\ \cline{2-2} & \underline{0{\cdot}4444,} \text{ \&c. of a yard.} \end{array}$$

4. Reduce 3*lb*. 4*oz*. 8*pwt*. 1*gr*. Troy, to the decimal of a *pound*. *Ans*. 3·36684027777, &c., of a lb.

5. Reduce 3*h.* 30*m.* 10*sec.* to the decimal of a day.
Ans. 0·145949074074, &c., of a day.

6. Reduce £3 5*s.* 0*d.* 2*far.* to the value of a £.
Ans. £3·252083333, &c.

7. Reduce 28 gallons of wine to the decimal of a hogshead. *Ans.* 0·4444, &c., of a hogshead.

8. Reduce 4*s* 6½*d.* to the decimal of a £.
Ans. £0·2270833, &c.

9. Reduce 18*s.* 3¾*d.* to the decimal of a £.
Ans. £0·915625.

10. Reduce 3 pecks, 5 quarts and 1 pint to the decimal of a bushel. *Ans.* 0·921875 of a bushel.

11. Reduce 11*hr.* 16*m.* 15*sec.* to the decimal of a day.
Ans. 0·469618055, &c., of a day.

12. Reduce 20 rods, 4 yards, 2 feet and 6 inches to the decimal of a furlong.
Ans. 0·521969696, &c., of a furlong.

13. Reduce 42*m.* 36*sec.* to the decimal of an hour.
Ans. 0·71, of an hour.

14. Reduce 30 days, 3 hours, 27 minutes, 30 seconds, to the decimal of a year of 365·24224 days.
Ans. 0·0827617, &c., of a year.

15. Reduce 5*hr.* 48*m.* 49·536*sec.* to the decimal of a day? *Ans.* 0·24224 of a day.

CASE II.

To find the proper value of denominate decimals.

Find the value of 0·778125 of a £.

OPERATION.

0·778125 of a £.
20 = shillings in £1.
15·562500 of a shilling.
12 = pence in 1s.
11250
5625
6·7500 of a penny.
4 = farthings in 1 penny.
3·0[illegible] farthings.

Which gives 15*s.* 6*d.* 3*far.*

EXPLANATION.

We first multiplied the decimal of a £ by 20, because 20 shillings make 1 pound; pointing off by the rule for decimals, we found 15*s.* and 0·5625 of a shilling. Then we multiplied this decimal of a shilling by 12, because 12 pence make 1 shilling; pointing off, we found 6*d.* and 0·75 of a penny, which being multiplied by 4, because 4 farthings make 1 penny, gave just 3 farthings.

By carefully considering the above operation, we deduce this

RULE.

Multiply the decimal by the number expressing how many of the next lower denomination make a unit of the denomination of the decimal; point off by the usual rule for decimals; multiply the decimal part, thus pointed off, *as* *before; and so continue to the lowest denomination; the several denominate values sought will appear at the left of the decimal point of the successive products.*

Repeat this Rule.

EXAMPLES

1. What is the value of 0·9075 of an acre?

OPERATION.

$$\begin{array}{l} 0{\cdot}9075 \\ \underline{\quad\;\; 4} = \text{roods in } 1A. \\ 3{\cdot}6300R. \\ \underline{\quad\;\; 40} = \text{rods in } 1R. \\ \underline{25{\cdot}2P.} \end{array}$$

Ans. 3*R.* 25·2*P.*

2. What is the value of £0·125? *Ans.* 2*s.* 6*d.*
3. What is the value of £0·66⅔? *Ans.* 13*s.* 4*d.*
4. What is the value of 0·375 of a hogshead of wine?
Ans. 23*gal.* 2*qt.* 1*pt.*
5. What is the value of 0·121212 of a year of 365 days?
Ans. 44*da.* 5*hr.* 49*m.* 1·632*sec.*
6. What is the value of 0·3355 of a pound avoirdupois?
Ans. 5*oz.* 5·888*dr.*
7. What is the value of 0·3322 of a ton?
Ans. 6*cwt.* 2*qr.* 16*lb.* 2·048*oz.*
8. What is the value of 0·2525 of a mile?
Ans. 2*fur.* 0*rd.* 4*yd.* 1*ft.* 2·4*in.*
9. What is the value of 0·345 of a £?
Ans. 6*s.* 10*d.* 3·2*far.*

10. What is the value of 0·121212 of a day?

Ans. 2*hr.* 54*m.* 32·7168*sec.*

11. What is the value of 0·3456 of a £?

Ans. 6*s.* 10*d.* 3·776*far.*

12. What is the value of 0·9875 of a £?

Ans. 19*s.* 9*d.*

13. What is the value of 0·24224 of a solar day?

Ans. 5*hr.* 48*m.* 49·536*sec.*

DUODECIMALS.

100. IN decimals we have seen that the figures decrease in a tenfold ratio, from the left towards the right.

In duodecimals, this decrement goes on in a twelvefold ratio.

The different denominations are the *foot* (*f.*) the *prime*, or inch (′), the *second* (″), the *third* (‴), the *fourth* (⁗), the *fifth* (′′′′′), and so on.

Thus, 7*f.*, 6′, 3″, 4‴, 5⁗, is read 7 feet, 6 primes, 3 seconds, 4 thirds, 5 fourths.

The accents used to distinguish the denominations below feet, are called *indices.*

Taking the foot for the unit, we have the following relations:

$1' = \frac{1}{12}$ of 1 foot.

$1'' = \frac{1}{12}$ of $\frac{1}{12}$ of 1 foot $= \frac{1}{144}$ of 1 foot.

$1''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 1 foot $= \frac{1}{1728}$ of 1 foot.

$1'''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 1 foot $= \frac{1}{20736}$ of 1 foot

&c. &c. &c. &c.

ADDITION AND SUBTRACTION OF DUODECIMALS

101. ADDITION AND SUBTRACTION of duodecimals, ar performed like addition and subtraction of other denominate numbers, remembering that 12 of any denominatio make one of the next greater denomination.

In decimals how do figures decrease from the left toward the right? In duodecimals how do they decrease? What are the different denominations of duodecimals What are the accents called which are used to distinguish the different denomination below the foot? How is addition and subtraction of duodecimals performed?

EXAMPLES.

(1.)

17*f.*	7′	8″
25*f.*	0′	2″
30*f.*	10′	11″
29*f.*	6′	6″
103*f.*	1′	3″

(2.)

365*f.*	1′	7″	9‴
521*f.*	10′	10″	11‴
605*f.*	8′	8″	1‴
731*f.*	3′	0″	8‴
2224*f.*	0′	3″	5‴

3. What is the sum of 3*f.* 6′ 4″, 8*f.* 3′ 4″, 9*f.* 1′ 3″, an 10*f.* 10′ 10″? *Ans.* 31*f.* 9′ 9″.

4. What is the sum of 100*f.* 8′ 8″, 135*f.* 0′ 1″, 65*f.* 9 2″, 45*f.* 3′ 3″, and 200*f.* 6′ 6″? *Ans.* 547*f.* 3′ 8″.

(5.)

From	87*f.*	3′	4″
Subtract	35*f.*	8′	9″
Remainder	51*f.*	6′	7″

(6.)

100*f.*	10′	10″
90*f.*	6′	3″
10*f.*	4′	7″

7. From 25*f.* 6′ 6″ subtract 18*f.* 9′ 10″. *Ans.* 6*f.* 8′ 8″.

8. From 100*f.* subtract 58*f.* 2′ 1″. *Ans.* 41*f.* 9′ 11″

MULTIPLICATION OF DUODECIMALS.

102. SUPPOSE we wish to multiply 14*f.* 7′ by 2*f.* 3′ we should proceed as follows:

$$\begin{array}{rrrl} & 14f. & 7' & \\ & 2f. & 3' & \\ \cline{2-3} & 3f. & 7' & 9'' \\ & 29f. & 2' & \\ \cline{2-4} \textit{Ans.} & 32f. & 9' & 9''=32f.+\tfrac{9}{12}\text{ of a foot}+\tfrac{9}{144}\text{ of a foot.} \end{array}$$

EXPLANATION.

We begin on the right hand, and multiply the multiplicand through, first by the primes of the multiplier, then by the feet of the multiplier, thus: $3'\times7'=\frac{3}{12}\times\frac{7}{12}=\frac{21}{144}$ of a foot, which is $21''=1'\,9''$; we write down the 9″, and reserve the 1′ for the next product; again, $14f.\times3'=14\times\frac{3}{12}=\frac{42}{12}$ of a foot, which is 42′; now adding in the 1′, which was reserved from the last product, we have $43'=3f.\,7'$, which we write down, thus finishing the first line of products.

Again, we have $2f.\times7'=2\times\frac{7}{12}=\frac{14}{12}$ of a foot, which is $14'=1f.\,2'$; we write the 2′ under the primes of the line above, and reserve the 1*f.* for the next product; $2f.\times14f.=28f.$, to which, adding in the 1*f.* reserved from the last product, we have 29*f.*, which we place underneath the feet of the line above. Taking the sum, we find 32*f.* 9′ 9″, for the answer.

From the above we infer, *that if we consider the index* *of the feet to be* 0, *then the denomination of each product will be denoted by the sum of the indices of the factors.*

Thus, *feet* by *feet*, produces *feet; feet* by *primes*, produces *primes; primes* by *primes*, produces *seconds*, &c.

Hence, to multiply a number consisting of feet, inches, seconds, &c., by another number consisting of like quantities, we have this

RULE.

Place the several terms of the multiplier under the corresponding ones of the multiplicand. Beginning at the right hand, multiply the several terms of the multiplicand by the several terms of the multiplier successively, placing the right-hand term of each of the partial products under its multiplier; then add the partial products together, observing to carry one for every twelve, both in multiplying and adding. The sum of the partial products will be the answer.

Repeat this Rule.

EXAMPLES.

1. What is the product of 3*f.* $7'$ $2''$ by 7*f.* $6'$ $3''$?

OPERATION.

			3*f.*	$7'$	$2''$
			7*f.*	$6'$	$3''$
			$10''$	$9'''$	$6''''$
	1*f.*	$9'$	$7''$	$0'''$	
	25*f.*	$2'$	$2''$		
Ans.	27*f.*	$0'$	$7''$	$9'''$	$6''''$
			17		

2. Multiply 7*f.* 8′ by 6*f.* 4′ 3″? *Ans.* 48*f.* 8′ 7″.

3 Multiply 6*f.* 9′ 7″ by 4*f.* 2′? *Ans.* 28*f.* 3′ 11″ 2‴.

4. What is the area of a marble slab, whose length is 7*f.* 3′, and breadth 2*f.* 11′? *Ans.* 21*f.* 1′ 9″.

5. How many square feet are contained in the floor of a hall 37*f.* 3′ long, by 10*f.* 7′ wide? *Ans.* 394*f.* 2′ 9″.

6. How many square feet are contained in a garden 100*f.* 6′ in length, by 39*f.* 7′ in width? *Ans.* 3978*f.* 1′ 6″.

7. How many yards of carpeting, one yard in width, will it require to cover a room 16*f.* 5′ by 13*f.* 7′?

Ans. 24*yd.* 6*f.* 11′ 11″.

REDUCTION OF CURRENCIES.

103. Before the adoption of Federal money in this country, accounts were generally kept in the denominations of English money. Different States considered the pound as having different values, as given in the following

TABLE.

$$\$1 \text{ in England} = 4s.\ 6d. = £\tfrac{9}{40}, \text{ called Sterling money.}$$

$$\$1 \text{ in } \left\{ \begin{array}{l} \text{South Carolina} \\ \text{Georgia} \end{array} \right\} = 4s.\ 8d. = £\tfrac{7}{30}, \text{ called Georgia currency.}$$

$$\$1 \text{ in } \left\{ \begin{array}{l} \text{Canada} \\ \text{Nova Scotia} \end{array} \right\} = 5s. = £\tfrac{1}{4}, \text{ called Canada currency.}$$

$$\$1 \text{ in } \left\{ \begin{array}{l} \text{New England States} \\ \text{Virginia} \\ \text{Kentucky} \\ \text{Tennessee} \end{array} \right\} = 6s. = £\tfrac{3}{10}, \text{ called New England currency.}$$

$$\$1 \text{ in} \left\{\begin{array}{l}\text{New Jersey}\\ \text{Pennsylvania}\\ \text{Delaware}\\ \text{Maryland}\end{array}\right\} = 7s.\ 6d. = £\tfrac{3}{8}, \text{ called Pennsylvania currency.}$$

$$\$1 \text{ in} \left\{\begin{array}{l}\text{New York}\\ \text{Ohio}\\ \text{North Carolina}\end{array}\right\} = 8s. = £\tfrac{2}{5}, \text{ called New York currency.}$$

How were accounts kept before the adoption of Federal money? Did all the States estimate the pound at the same value? What fraction of a £ is $1 in Sterling money? What part of a £ is $1 in Georgia currency? What part of a £ is $1 Canada currency? What part of a £ is $1 New England currency? What part in Pennsylvania currency? What part in New York currency?

CASE I.

104. To reduce Federal money to pounds, shillings, and pence, we obviously have this

RULE.

Multiply the sum in Federal money by the value of $1 *expressed in the fraction of a pound, as given in the above Table; the product will be pounds. If there are decimals of a pound, they must be reduced to shillings and pence by Rule under* ART. **99.**

What is the fraction by which we multiply Federal money to reduce it to Sterling money? What fraction do we multiply to reduce it to Georgia currency? What is the fraction for Canada currency? What for New England currency? What for Pennsylvania currency? What for New York currency? If in the product there are decimals of a pound, how do you dispose of them?

EXAMPLES.

1. Reduce $100·20 to the different currencies, as given in the *preceding* Table.

	£	s.	d.	
Ans. $100·20 =	22	10	10⅘	Sterling money.
	23	7	7⅕	Georgia currency.
	25	1	0	Canada currency.
	30	1	2⅖	New England currency
	37	11	6	Pennsylvania currency.
	40	1	7⅕	New York currency.

2. Reduce $37·37 to the different currencies.

	£	s.	d.	
Ans. $37·37 =	8	8	1·98	Sterling money.
	8	14	4·72	Georgia currency.
	9	6	10·2	Canada currency.
	11	4	2·64	New England currency
	14	0	3·3	Pennsylvania currency.
	14	18	11·52	New York currency.

3. Reduce $1000 to equivalent values in the differen currencies.

	£	
Ans. $1000 =	225	Sterling money.
	233 6*s.* 8*d.*	Georgia currency.
	250	Canada currency.
	300	New England currency.
	375	Pennsylvania currency.
	400	New York currency.

CASE II.

105. To reduce a sum in either of the above currencie to Federal money.

It is obvious, that by inverting the fractions which ex *press the value of $1 in pounds, as given in the precedin* table, we shall obtain the value of £1 in dollars. Consequently, we deduce this

RULE.

I. Reduce the shillings and pence, if any, to a decimal of a pound, by Rule under Art. **99.**

II. Multiply the pounds and decimals, if any, by the fractions of the preceding table, after inverting them; the products will be in dollars and decimals of a dollar.

By what fraction must we multiply Sterling money to reduce it to Federal money? What fraction do we multiply by to reduce Georgia currency to Federal money? By what do we multiply to reduce Canada currency? By what to reduce New England currency? By what to reduce Pennsylvania currency? By what to reduce New York currency?

EXAMPLES.

1. Reduce £75 15*s*. 6*d*. of the respective currencies mentioned in the preceding table, to Federal money.

£75 15*s*. 6*d*.=£75·775, which multiplied by the respective fractions $\frac{40}{9}$, $\frac{30}{7}$, $\frac{4}{1}$, $\frac{10}{3}$, $\frac{8}{3}$, and $\frac{5}{2}$, gives the following answer:

***Ans.* £75 15*s*. 6*d*.**
Sterling money =\$336·77$\frac{7}{9}$.
Georgia currency = 324·75.
Canada currency = 303·10.
New England currency = 252·58$\frac{1}{3}$.
Pennsylvania currency = 202·06$\frac{2}{3}$.
New York currency = 189·43$\frac{3}{4}$.

2. Reduce £80 5*s*. 3*d*. of the different currencies to Federal money.

Ans. £80 5*s.* 3*d.*
Sterling money =$356·722$\frac{2}{9}$.
Georgia currency = 343·982$\frac{1}{7}$.
Canada currency = 321·05.
New England currency = 267·541$\frac{2}{3}$.
Pennsylvania currency = 214·033$\frac{1}{3}$.
New York currency = 200·656$\frac{1}{4}$.

3. Reduce £1000 of the different currencies to Federal money.

Ans. £1000.
Sterling money =$4444·444$\frac{4}{9}$.
Georgia currency = 4285·714$\frac{2}{7}$.
Canada currency = 4000.
New England currency = 3333·333$\frac{1}{3}$.
Pennsylvania currency = 2666·666$\frac{2}{3}$.
New York currency = 2500.

106. The following are the rates at which some of the foreign coins are estimated at the custom-houses of the United States:

English £	$4·84.
Livre of France	$0·18$\frac{1}{2}$.
Franc of do.	$0·18$\frac{3}{4}$.
Silver Rouble of Russia	$0·75.
Florin or Guilder of the United Netherlands	$0·40.
Mark Banco of Hamburg	$0·35.
Real of Plate of Spain	$0·10.
Real of Vellon of do.	$0·05.
Milrea of Portugal	$1·12$\frac{1}{2}$.
Tale of China	$1·48.
Pagoda of India	$1·84.
Rupee of Bengal	$0·50.
Specie dollar of Sweden and Norway . .	$1·06.
Specie dollar of Denmark	$1·05.

Thaler of Prussia and N. States of Germany	$0·69.
Florin of Austrian Empire and City of Augsburg	$0·48½.
Lira Lombardo-Venetian Kingdom and of Tuscany	$0·16.
Ducat of Naples	$0·80.
Ounce of Sicily	$2·40.
Pound of British Provinces, Nova Scotia, New Brunswick, Newfoundland, and Canada .	$4·00.
Rix-dollar of Bremen	$0·78¾.
Thaler of Bremen	$0·71.
Mil-reis of Madeira	$1·00.
" of Azores	$0·83⅓.
Rupee of British India	$0·44½.

RULE OF THREE.

107. The quotient arising from dividing one quantity by another of the same *kind* or *denomination*, is called a *ratio*.

Thus, the ratio of

$$12 \text{ to } 2=\tfrac{12}{2}=6.$$
$$12 \text{ to } 3=\tfrac{12}{3}=4.$$
$$12 \text{ to } 4=\tfrac{12}{4}=3.$$
$$12 \text{ to } 6=\tfrac{12}{6}=2.$$
$$12 \text{ to } 12=\tfrac{12}{12}=1.$$

Hence, we see that the ratio of two quantities shows *how many times* greater the one is than the other. It is therefore evident, that there cannot exist a ratio between two quantities of different denominations. There is no ratio between 12 feet and 3 pounds, for we cannot say how many times 12 feet is greater than 3 pounds. But there is a ratio between 12 feet and 3 feet, which is

$$\frac{12\,ft.}{3\,ft.}=4.$$

There is the same ratio between 12 pounds and 3 pounds. The ratio is itself an abstract number; it is not a denominate number. The ratio of 12 feet to 3 feet is 4 units simply; it is neither 4 feet nor 4 pounds, but simply 4 times 1; showing that 12 feet is 4 times as great as 3 feet. In this way we find

The ratio of 10 yards to 5 yards $=\frac{10}{5}=2.$
" 8 inches to 4 inches $=\frac{8}{4}=2.$
" 7 ounces to 3 ounces $=\frac{7}{3}=2\frac{1}{3}.$
" 5 bushels to 2 bushels $=\frac{5}{2}=2\frac{1}{2}.$
" 7 rods to 4 rods $=\frac{7}{4}=1\frac{3}{4}.$
" 9 cords to 4 cords $=\frac{9}{4}=2\frac{1}{4}.$
" 40 acres to 18 acres $=\frac{40}{18}=\frac{20}{9}=2\frac{2}{9}.$

When the ratio of two quantities is the same as the ratio of two other quantities, the four quantities are in *proportion*. Thus, the ratio of 8 yards to 4 yards, is the same as the ratio of 12 dollars to 6 dollars; therefore, there is a proportion between 8 yards, 4 yards, 12 dollars, and 6 dollars.

The usual method of denoting that four terms are in proportion, is by means of points, or dots. Thus, the above proportion is written

8 yards : 4 yards : : 12 dollars : 6 dollars;

in which two dots are placed between the first and second terms, and between the third and fourth; and four dots between the second and third. The proportion is read

8 yards is to 4 yards as 12 dollars is to 6 dollars.

Of these four terms, the first and fourth are called *extremes;* the second and third are called *means.*

Since in a proportion the quotient of the first term divided by the second, is equal to the quotient of the third term divided by the fourth, we have, using the above proportion,

$$\left.\frac{8\text{ yards}}{4\text{ yards}}=\frac{12\text{ dollars}}{6\text{ dollars}}\right\}\text{ or, which is still more simply expressed, }\left\{\frac{8}{4}=\frac{12}{6}.\right.$$

If we reduce these fractions to a common denominator, (**Art. 40**,) they will become

$\frac{8\times6}{4\times6}=\frac{12\times4}{6\times4}$, or, omitting the common denominator 4×6, which is in effect multiplying each fraction by 4×6, we have 8×6 or $48=12\times4$ or 48; that is, *the product of the extremes is equal to the product of the means.*

$$\text{Again, }\frac{8\times6=48}{12}=4,\text{ and }\frac{8\times6=48}{4}=12.$$

Hence, if the product of the extremes be divided by either mean, the quotient will be the other mean.

$$\text{Again, }\frac{12\times4}{8}=6,\text{ and }\frac{12\times4}{6}=8.$$

Hence, if the product of the means be divided by either extreme, the quotient will be the other extreme.

From the above properties, we see that if any *three* of the four terms which constitute a proportion are given, the remaining term can be found.

108. The method of finding the fourth term of a proportion, when *three* terms are given, constitutes the **Rule of Three.**

What is the quotient arising from dividing one number by another of the same kind called? What is the ratio of 12 to 2? Of 12 to 3? Of 12 to 4? What does the ratio of two quantities show? Can a ratio exist between two quantities of different denominations? Is there a ratio between 12 feet and 3 pounds? Can the ratio be a denominate number? How are four quantities related when the ratio of the first to the second is the same as the ratio of the third to the fourth? Which are called extremes? Which are called means? To what is the product of the extremes equal? If the product of the extremes be divided by one of the means, what will the quotient be? How many terms of a proportion must be known in order to find the others?

1. Let us endeavor to find the value of 24 yards of cloth, on the supposition that 8 yards are worth $12.

It is obvious that the value sought must be as many times greater than $12 as 24 yards is greater than 8 yards. Hence, there is the same ratio between $12 and the *value sought*, as there is between 8 yards and 24 yards. Consequently, we have this proportion:

8 yards : 24 yards : : $12 : *value sought.*

Taking the product of the means, we have $24 \times 12 = 288$. This, divided by the first term, gives $\frac{288}{8} = 36$ for the fourth term sought, which must be of the same kind as the third term; therefore, $36 is the value of 24 yards.

NOTE.—When we take the product of the means we do not multiply the 24 yards by 12 dollars, but simply multiply 24, the number denoting the yards, by 12, the number denoting the dollars. The product, 288, is neither yards nor dollars, but 288 units. When we divide this product by the first term of the proportion, we do not divide by 8 yards, but simply divide by 8, the number denoting the yards. The quotient, 36, gives the fourth term of the proportion; and since the fourth term is of the same denominate value as the third term, our fourth term, or answer, must be 36 dollars.

2. What will 312 pounds of coffee cost, if 25 pounds cost $3·25?

In this example, the ratio of 25 pounds to 312 pounds, *is the* same as the ratio of $3·25 to the number of dollars *sought*. Hence, 25 pounds : 312 pounds : : $3·25 : the answer.

```
                                    312
                                    ———
                                    650
                                   325
                                  975
                                 —————
                           25)1014·00($40·56
                              100
                              ———
                                140
                                125
                                ———
                                 150
                                 150
                                 ———
```

Here we first multiply the means together; we then divide the product by the first term.

Since there is a ratio between the third and fourth terms, it follows that they must be of the same denominate value. Hence, of the three quantities given, we may always take for the third term of our proportion the quantity which is of the same kind as the answer required; then, if the answer sought is to be greater than this third term, the second term must exceed the first; but if the answer sought is to be less than this third term, then the second term must be less than the first.

109. From what has been said and done, we deduce this first form for the

RULE OF THREE.

I. Form a proportion by placing for the third term, the quantity which is of the same kind as the answer sought; the two remaining quantities must be taken for the first and second terms, observing to take the larger of the two quantities for the second term, when the answer sought is to exceed the third term; but to take the smaller of the two quantities for *the second term, when the answer is to be less than the third term.*

II. Having written the three terms of the proportion, or, as usually expressed, having stated the question, then multiply the second and third terms together, and divide the product by the first term.

NOTE.—Since there is a ratio between the first and second terms, they must be reduced to the same denominate value. Also, the third term must be reduced to its lowest denomination; then the quotient found by dividing the product of the means by the first term, will be of the same denomination as the third term.

In stating questions in the Rule of Three, which quantity must be taken for the third term? Of the two remaining quantities, which is to be taken for the second term? After the question is stated, how do you proceed to find the answer? Is it ever necessary to make any reduction in the terms before multiplying and dividing? What are these reductions? The answer when found, will be of the same name as which term?

EXAMPLES.

1. What is the cost of 6 cords of wood, at $7 for 2 cords?

2 cords : 6 cords : : $7 : *Ans.*
6
2)42
Ans. $21

2. What will 9 pair of shoes cost, if 5 pair cost £2 2*s.* 6*d.*?

5 pair : 9 pair : : £2 2*s.* 6*d.*
When reduced, 5 pair : 9 pair : : 510*d.*
9
5)4590
Ans. 918d.=£3 16s. 6*d.*

3. If there are 9 weeks in 63 days, how many weeks are there in 365 days?

63 days : 365 : : 9 weeks.
9
63)3285(52$\frac{9}{63}$=52$\frac{1}{7}$ weeks. *Ans.*
315
135
126
9

4. If a railroad car goes 17 miles in 45 minutes, how far will it go in 5 hours?

45 minutes : 5 hours : : 17 miles.
or 45 " : 300 minutes : : 17 "
17
2100
300
45)5100(113$\frac{1}{3}$ miles. *Ans.*
45
60
45
150
135
15

5. If $100 will gain $7 in one year, how long will it require to gain $100? *Ans.* 14$\frac{2}{7}$ years.

6. If 3 paces or common steps of a person is equal to 2 yards, how many yards will 480 paces make?

Ans. 320 yards.

7. If 15 men can raise a wall of masonry 12 feet in one week, how many will be necessary to raise it 20 feet in the same time? *Ans.* 25 men.

8. *If 5 tons of* coal, of 2000 pounds each, will last 3½

months of 30 days each, how much will be consumed in 3 weeks, or 21 days? *Ans.* 1 ton, or 2000 pounds.

9. If 9½ bushels of wheat make 2 barrels of flour, how many bushels will be required to make 13 barrels? *Ans.* 61¾ bushels.

10. If a steamboat of 242 feet in length move 15 miles in one hour, how many seconds will it require to move its own length? *Ans.* 11 seconds.

11. If a steamboat of 242 feet in length move 15 miles an hour, how many times its own length will it move in 11 hours? *Ans.* 3600 times.

12. A reservoir has a pipe capable of discharging 30 gallons in one minute, what time will be necessary to discharge 15 hogsheads? *Ans.* 31½ minutes.

13. If a man can mow 9 acres of grass in 3½ days of 10 hours each, how long will it require for him to mow 21 acres? *Ans.* 8⅙ days.

14. If 100 pounds of galena, or lead ore, yield 83 pounds of pure metal, how much pure metal will 7 tons of galena produce, if we reckon 2240 pounds to the ton? *Ans.* 13014⅖ pounds.

15. If 12 barrels of flour are worth $54, what is the value of 42 barrels at the same rate?

```
12 barrels : 42 barrels : : $54
                             42
                            108
                           216
                       12)2268(189 dollars. Ans.
                           12
                           106
                            96
                            108
                            108
```

In this example, it is obvious that 2 times 12 barrels would be worth 2 times \$54; 3 times 12 barrels would be worth 3 times \$54; 4 times 12 barrels would be worth 4 times \$54. These ratios 2, 3, 4, may be expressed by $\frac{2\times12}{12}=\frac{24}{12}, \frac{3\times12}{12}=\frac{36}{12}, \frac{4\times12}{12}=\frac{48}{12}$; and in a similar manner, the ratio of 42 barrels to 12 barrels is $\frac{42}{12}$.

If we multiply \$54 by this ratio, it will give the value of 42 barrels. The operation may be expressed thus: $\$54\times\frac{42}{12}$. We may now simplify this expression as by ART. **39.** Thus, dividing the denominator 12, and the numerator 42, each by 6, the expression becomes

$$\$54\times\frac{\overset{7}{\cancel{42}}}{\underset{2}{\cancel{12}}}, \text{ or } \$54\times\frac{7}{2}.$$

Cancelling the denominator 2 against a corresponding factor of the numerator 54 $(=\frac{54}{1})$, we have

$$\overset{27}{\cancel{\$54}}\times\frac{7}{\cancel{2}}, \text{ or } \$27\times7=\$189. \textit{ Ans.}$$

16. What will 84 bushels of apples cost, if 14 bushels are worth \$6·75 ?

The ratio of 84 bushels to 14 bushels is $\frac{84}{14}$. Now, multiplying \$6·75 by this ratio, we have

$$\$6{\cdot}75\times\frac{84}{14}.$$

Dividing 84 of numerator and 14 of the denominator each by 7, we obtain

$$\$6{\cdot}75\times\frac{\overset{12}{\cancel{84}}}{\underset{2}{\cancel{14}}}, \text{ or } \$6{\cdot}75\times\frac{12}{2}.$$

Again, dividing 12 of numerator and 2 of denominator each by 2,

$$\$6{\cdot}75 \times \frac{\overset{6}{\cancel{12}}}{\cancel{2}} \text{ or, } \$6{\cdot}75 \times 6 = \$40{\cdot}50. \ \textit{Ans.}$$

From these two examples, we see that the Rule of Three may be given in the following simple form:

RULE OF THREE.

Of the three quantities which are given, one will always be of the same kind as the answer sought; this quantity will be the third term. Then, if by the nature of the question, the answer is required to be greater than the third term, divide the greater of the two remaining quantities by the less, for a ratio; but if the answer is required to be less than the third term, then divide the less of the two remaining quantities by the greater, for a ratio. Having obtained the ratio, multiply the third term by it, and it will give the answer in the same denomination as is the third term.

NOTE.—Before obtaining the ratio, by means of the first two terms, we must reduce them to like denominations.

17. If 200 sheep yield 650 pounds of wool, how many pounds will 825 sheep yield?

In this example, the answer is required to be in pounds; we therefore take 650 pounds for the third term. The ratio of 825 sheep to 200 sheep is $\frac{825}{200}$. Hence we have

$$650\textit{lb.} \times \tfrac{825}{200}.$$

Cancelling, we have

$$650lb.\times\frac{\overset{33}{\cancel{825}}}{\underset{8}{\cancel{200}}}\text{ or }650lb.\times\frac{33}{8}.$$

Again, cancelling, we have

$$325lb.\times\frac{33}{\underset{4}{\cancel{8}}}=\frac{325\times33}{4}=2681\tfrac{1}{4}lb.\ \mathit{Ans.}$$

18. If $\frac{11}{13}$ of a pound of sugar cost $\frac{23}{26}$ of a shilling, how much will $\frac{9}{23}$ of a pound cost?

In this example, our third term is $\frac{23}{26}$ of a shilling. And since $\frac{9}{23}$ of a pound is less than $\frac{11}{13}$, we must obtain our ratio by dividing $\frac{9}{23}$ by $\frac{11}{13}$, which gives $\frac{9}{23}\times\frac{13}{11}$. Multiplying the third term by this ratio, we have $\frac{23}{26}$ of a shilling $\times\frac{9}{23}\times\frac{13}{11}$. To reduce this with the least labor, we must resort to the method of cancelling. Thus, cancelling the 23, which occurs in both numerator and denominator, also 13 of the numerator against a corresponding factor of the 26 of the denominator, our expression will become $\frac{1}{2}$ of a shilling $\times\frac{9}{1}\times\frac{1}{11}=\frac{9}{22}$ of a shilling.

NOTE.—This method of cancelling should be used when the nature of the question will admit, since it will always simplify the operation

19. If a tree 38 feet 9 inches in height, give a shadow of 49 feet 2 inches, how high is that tree which, at the same time, casts a shadow of 71 feet 7 inches?

In this example, our third term is the height of the first tree, which is 38 feet 9 inches $=38\frac{3}{4}$ feet $=\frac{155}{4}$ feet: our ratio will be obtained by dividing 71 feet 7 inches $=71\frac{7}{12}$ feet $=\frac{859}{12}$ feet, by 49 feet 2 inches $=49\frac{1}{6}$ feet $=\frac{295}{6}$ feet: *which thus becomes* $\frac{859}{12}\times\frac{6}{295}$. Multiplying the third term by this ratio, we have $\frac{155}{4}$ feet $\times \frac{859}{12} \times \frac{6}{295}$. Cancelling 6 of the numerator against 6, a factor of the 12 of the denominator, also cancelling 5, a factor of 155 of the numerator, against 5, a factor of 295 of the denominator, we get $\frac{31}{4}$ feet $\times \frac{859}{2} \times \frac{1}{59} = \frac{26629}{472} = 56\frac{197}{472}$ feet, for the answer.

20. If $3\frac{1}{2}$ pounds of coffee cost $2\frac{1}{3}$ shillings, how much will $10\frac{1}{6}$ pounds cost?

In this example, $2\frac{1}{3} = \frac{7}{3}$ shillings must be our third term; and since $10\frac{1}{6} = \frac{61}{6}$ pounds must cost more than $3\frac{1}{2} = \frac{7}{2}$ pounds, we must divide $\frac{61}{6}$ by $\frac{7}{2}$ for the ratio; making it $\frac{61}{6} \times \frac{2}{7}$. Multiplying the third term by this ratio, we obtain $\frac{7}{3}$ shillings $\times \frac{61}{6} \times \frac{2}{7}$; which, after cancelling, becomes $\frac{1}{3}$ of a shilling $\times \frac{61}{3} = \frac{61}{9}$ shillings $= 6\frac{7}{9}$ shillings.

21. Gave \$72 for 11 barrels of fish. How much will 88 barrels cost at the same rate? *Ans.* \$576.

22. If $43\frac{1}{3}$ pounds of cheese cost \$2·20, what will $216\frac{2}{3}$ pounds cost at the same rate? *Ans.* \$11.

23. If I pay \$3·90 for sawing 7 cords of wood, how much ought I to give for sawing $23\frac{1}{3}$ cords? *Ans.* \$13.

24. If $\frac{3}{10}$ of a ship is worth \$2853, what is the whole worth?

The ratio of the whole ship, or $\frac{10}{10}$, to $\frac{3}{10}$, is $\frac{10}{3}$. Hence,

$$\$2853 \times \tfrac{10}{3} = \$951 \times 10 = \$9510 \ \textit{Ans.}$$

25. If $\frac{4}{13}$ of my income is \$533, what is my whole income? *Ans.* \$1732·25.

26. A person failing in business, finds that he owes \$7560, and that he only has \$3100 to pay it with. How much can he pay to that creditor whose claim is \$756? *Ans.* \$310.

27. If it require $5\frac{1}{2}$ bushels of wheat to make one barrel of flour, how many bushels will it require for 100 barrels of flour? Ans. 550 bushels.

28. If 7 barrels of flour are sufficient for a family 6 months, how many barrels will they require for 11 months? *Ans.* $12\frac{5}{6}$ barrels.

29. If it take 25 yards of carpeting, a yard wide, to cover a certain floor, how many yards of $\frac{3}{4}$ carpeting would be necessary to cover the same floor? *Ans.* $33\frac{1}{3}$ yards.

30. If a person travel 8 miles in 10 hours, how far will he travel in 5 days, by traveling 8 hours each day? *Ans.* 32 miles.

31. If 35 pounds of feathers cost $15, what will 100 pounds cost at the same rate? *Ans.* $\$42{\cdot}85\frac{5}{7}$.

32. If a man perform a certain piece of work in 18 days, when he works 8 hours per day, how many days will he require if he work 10 hours each day? *Ans.* 14 days, 4 hours.

33. If a piece of board 12 inches wide and 12 inches long make one square foot, how many inches of length must be taken from a board 15 inches wide to make a square foot? *Ans.* $9\frac{3}{5}$ inches.

34. If 8 men can mow a field in 5 days, in how many days can 5 men do the same? *Ans.* 8 days.

35. If $27\frac{1}{2}$ yards of cloth cost $60, how many yards can I buy for $100? *Ans.* $45\frac{5}{6}$ yards.

36. If $27\frac{1}{2}$ yards of cloth cost $60, what will $45\frac{5}{6}$ yards cost? *Ans.* $100.

37. If $\frac{5}{8}$ of a ship is worth $9000, what is her whole value?

The whole ship being a unit, or $\frac{8}{8}$, we have the ratio $\frac{8}{5}$; hence, the answer is $\$9000\times\frac{8}{5}=\$14400$.

38. If $\frac{3}{16}$ of a city lot is sold for $500, what would $\frac{7}{16}$ of the same lot sell for at the same rate? *Ans.* $\$1166\frac{2}{3}$.

39. Admitting that the earth moves in its orbit about

the sun, a distance of 597000000 miles, in 365 days 6 hours, how far on an average does it move in each hour?

Ans. 68104$\frac{56}{1461}$ miles.

40. The equatorial portions, by the diurnal rotation of the earth, moves about 24900 miles each day? How far is that in each hour? *Ans.* 1037½ miles.

41. If it require 10 years of 365¼ days for light to pass from a fixed star to the earth, how many miles distant is it, on the supposition that light moves 192000 miles in one second? *Ans.* 60590592000000 miles.

42. If by a leak of a ship ⅗ enough water run in, in 4 hours, to sink her, how long can she survive?

Ans. 6*hr.* 40*m.*

43. If I pay $25 for the masonry of 4000 bricks, how much ought I to pay for the work which requires 100000 bricks? *Ans.* $625.

44. If a steam-ship require 14 days to sail a distance of 3000 miles, what time, at the same rate of sailing, would she require to sail 24900 miles?

Ans. 116 days 4⅘ hours.

45. Admitting the diameter of the earth to be 8000 miles, and the highest mountain to be 5 miles, what elevation must be made on the globe of 16 inches diameter to represent accurately the height of such mountain?

Ans. $\frac{1}{100}$ of an inch.

46. If $100 in 12 months bring an interest of $7, how much will be the interest of $100 for 8 months?

Ans. $4·66⅔.

47. If the interest of $100 for 12 months is $7, what will be the interest of $75 for the same time?

Ans. $5·25.

48. If in 12 months the interest of $100 is $7, how long must $100 be on interest to gain $10?

Ans. $17\frac{1}{7}$ months.

49. If a glacier of 60 miles in length move 50 inches per annum, in what time will it move its whole length?

Ans. 76032 years.

50. If a staff of 10 feet in length give a shadow of 15 feet, how high is that tree whose shadow measures 90 feet? *Ans.* 60 feet.

51. Suppose sound to move 1100 feet in a second; how many miles distant is a cloud, in which lightning is observed 16 seconds before the thunder is heard, no allowance being made for the motion of light? *Ans.* $3\frac{1}{3}$ miles.

52. If it require 30 yards of carpeting which is $\frac{3}{4}$ of a yard wide to cover a floor, how many yards of carpeting which is $1\frac{1}{4}$ yards wide will be necessary to cover the same floor? *Ans.* 18 yards.

53. If the earth move through 12 signs, or 360° in $365\frac{1}{4}$ days, how far will she move in a lunar month of $29\frac{1}{2}$ days?

Ans. $29\frac{37}{487}$ degrees.

54. Suppose a steamboat capable of making 15 miles each hour, to move with a current whose velocity is $2\frac{1}{2}$ miles per hour, what will be the whole distance made during $13\frac{1}{2}$ hours? And what distance will the boat move in the same time against the same current?

Ans. { With the current, $236\frac{1}{4}$ miles.
Against the " $168\frac{3}{4}$ "

55. If the magnetic influence move through the telegraphic wires at the rate of 200000 miles in one second of time, how many times could it pass around the world in one second, allowing the circumference of the earth to be *24899 miles?* Ans. $8\frac{808}{24899}$ times.

56. If A can do a piece of work in 7 days, and B can do it in 8 days, what part of it can both do in 3½ days?

Ans. $\frac{15}{16}$ of it.

57. A reservoir, whose capacity is 1000 hogsheads, has a supply pipe by means of which it receives 300 gallons each hour; it also has two discharging pipes, the first of which discharges ⅚ of a gallon each minute, the second discharges 1¼ gallons per minute. The reservoir being empty, in what time will it be filled if the supply pipe alone is opened? In what time, if the supply pipe and the first discharging pipe are opened? In what time, if the supply pipe and the second discharging pipe? And in what time, if all three are opened?

Ans.	Supply pipe only opened,	210 hours =	8¾ days.
	" " and 1st dis. pipe,	252 " =	10½ "
	" " " 2d, " "	280 " =	11⅔ "
	" " 1st and 2d " "	360 " =	15 "

COMPOUND PROPORTION.

110. When the quantity required depends upon more than three terms, the operation of finding it is called the *Rule of Compound Proportion.*

Suppose we have the following example:

If 6 men can mow 30 acres of grass in 5 days, by working 8 hours each day, how many acres can 4 men mow in 9 days of 10 hours each?

Had the number of days, as well as hours in each day, *been the* same in both cases, the question would have been *equivalent* to the following:

If 6 men mow 30 acres of grass, how many acres will 4 men mow?

It is evident the number of acres sought would be the same fractional part of 30 acres that 4 men is of 6 men; that is, the quantity required is

$\frac{4}{6}$ of 30 acres.

If, now, we take into account the number of days, still supposing the number of hours in each day to remain the same in both cases, our question would become:

If $\frac{4}{6}$ of 30 acres can be mowed in 5 days, how much can be mowed in 9 days?

The answer in this case is obviously

$\frac{9}{5}$ of $\frac{4}{6}$ of 30 acres.

Now, taking into account the number of hours each day, our question will become as follows:

If $\frac{9}{5}$ of $\frac{4}{6}$ of 30 acres can be mowed in a certain time, when 8 hours are reckoned to each day, how much could be mowed when 10 hours are reckoned to each day?

This leads to the following final result:

$\frac{10}{8}$ of $\frac{9}{5}$ of $\frac{4}{6}$ of 30 acres.

By cancelling, we reduce this last expression to 45 acres. From the above work we see that questions of Compound Proportion may be solved by the following

RULE.

Among the quantities given, there will be but one like the answer, which one we will call the odd quantity. The other quantities will appear in pairs or couplets. Form ratios out of each couplet in the same manner as in the Rule of Three; then multiply all the ratios and the odd quantity together, and this will give the answer in the same denomination as the odd quantity.

NOTE.—Before forming ratios from the couplets, they must be reduced to the same denominate value.

EXAMPLES.

1. If a person travel 300 miles in 17 days, traveling only 6 hours each day, how many miles could he have gone in 15 days, by traveling 10 hours each day?

In this example, the answer is required in miles, therefore our odd term is 300 miles?

The first couplet consists of days; and since in 15 days, other things being the same, he could not travel as far as in 17 days, we must divide 15 by 17, which gives $\frac{15}{17}$ for the first ratio.

The second couplet consists of hours; and since in 10 hours he could travel farther than in 6 hours, we must divide 10 by 6, which gives $\frac{10}{6}$ for the second ratio.

Multiplying these two ratios and the odd term together, we get 300 miles $\times\frac{15}{17}\times\frac{10}{6}$. Cancelling the 6 of the denominator against 6, a factor of 300 $(=\frac{300}{1})$ of the numerator, we have $50\times\frac{15}{17}\times\frac{10}{1}=441\frac{3}{17}$ miles, for the answer.

2. If a marble slab 10 feet long, 3 feet wide, and 3 inches thick, weigh 400 pounds, what will be the weight of another slab, of the same marble, whose length is 8 feet, width 4 feet, and thickness 5 inches?

In this example, the answer is required to be given in pounds; therefore 400 pounds is the odd term. The first couplet consists of the lengths; and since 8 feet in length will give less weight than 10 feet, we must divide 8 by 10, which gives $\frac{8}{10}$ for the first ratio.

The second couplet consists of the widths; and since 4 feet wide will give more weight than 3 feet, we must divide 4 by 3, which gives $\frac{4}{3}$ for the second ratio.

The third couplet consists of thicknesses; and since 5 inches thick will give more weight than 3 inches, we must divide 5 by 3, which gives $\frac{5}{3}$ for the third ratio.

Multiplying the odd term and these ratios together, we get 400lbs. $\times\frac{8}{10}\times\frac{4}{3}\times\frac{5}{3}$. Cancelling the 10 of the denominator against a part of the 400 of the odd term numerator, we get 40lbs. $\times\frac{8}{1}\times\frac{4}{3}\times\frac{5}{3}=\frac{6400}{9}=711\frac{1}{9}$ pounds, for the answer.

3. 500 men, working 12 hours each day, have been employed 57 days to dig a canal of 1800 yards long 7 yards wide, and 3 yards deep; how many days must 860 men, working 10 hours each day be emlpoyed in digging another canal of 2900 yards long, 12 yards wide, and 5 yards deep, in a soil which is 3 times as difficult to excavate as the first?

In this example, the odd term is 57 days.

The different ratios will be as follows:

$\frac{500}{860}=\frac{25}{43}$ ratio of the men.

$\frac{12}{10}=\frac{6}{5}$ ratio of the hours.

$\frac{2900}{1800}=\frac{29}{18}$ ratio of lengths of the canals.

$\frac{12}{7}=$ ratio of widths of the canals.

$\frac{5}{3}=$ ratio of depths of the canals.

$\frac{3}{1}=$ ratio of the difficulty in excavation.

Multiplying successively these ratios and the odd term, we have

$$57 \text{ days}\times\frac{25}{43}\times\frac{6}{5}\times\frac{29}{18}\times\frac{12}{7}\times\frac{5}{3}\times\frac{3}{1}.$$

This becomes, after cancelling factors,

$$19 \text{ days}\times\frac{5}{43}\times\frac{6}{1}\times\frac{29}{1}\times\frac{2}{7}\times\frac{5}{1}\times\frac{1}{1}=549\tfrac{51}{301} \text{ days}.$$

4. 15 men, working 10 hours each day, have employed 18 days to build 450 yards of stone fence; how many men, working 12 hours each day, for 8 days, will be requisite to build 480 yards of similar fence? *Ans.* 30 men.

5. If it require 1200 yards of cloth $\frac{5}{4}$ wide to clothe 500 men, how many yards which is $\frac{7}{8}$ wide will it take to clothe 960 men? *Ans.* 3291$\frac{3}{7}$ yards.

6. If 8 men will mow 36 acres of grass in 9 days, by working 9 hours each day, how many men will be required to mow 48 acres in 12 days, by working 12 hours each day? *Ans.* 6 men.

7. If 11 men can cut 49 cords of wood in 7 days, when they work 14 hours per day, how many men will it take to cut 140 cords in 28 days, by working 10 hours each day? *Ans.* 11 men.

8. If 12 ounces of wool make 2$\frac{1}{2}$ yards of cloth, that is 6 quarters wide, how many pounds of wool will it take for 150 yards of cloth, 4 quarters wide? *Ans.* 30 pounds.

9. If the wages of 6 men for 14 days be 84 dollars, what will be the wages of 9 men for 16 days? *Ans.* $144.

10. If 100 men in 40 days of 10 hours each, build a wall 30 feet long, 8 feet high, and 2 feet thick, how many men must be employed to build a wall 40 feet in length, 6 feet high, and 4 feet thick, in 20 days, by working 8 hours each day? *Ans.* 500 men.

11. In how many days, working 9 hours a day, will 24 men dig a trench 420 yards long, 5 yards wide, and 3 yards deep, if 248 men, working 11 hours a day, in 5 days, dig a trench 230 yards long, 3 yards wide, and 2 yards deep? *Ans.* 288$\frac{52}{207}$ days.

12. Suppose that 50 men, by working 5 hours each day, can dig, in 54 days, 24 cellars, which are each 36 feet long, 21 feet wide, and 10 feet deep, how many men would be required to dig, in 27 days, 18 cellars, which are each 48 feet long, 28 feet wide, and 9 feet deep, provided they work only 3 hours each day? *Ans.* 200 men.

PRACTICE.

111. PRACTICE is a short method of finding the answer to such questions in the *Rule of Three* as have a unit for their first term. So named, because in the ordinary practical business of life very frequent use is made of it.

As an example, suppose one bushel of apples to be worth 50 cents, what is the value of $18\frac{1}{2}$ bushels?

Had the apples been worth $1 per bushel, it is plain that $18\frac{1}{2}$ bushels would have been worth $$18\frac{1}{2}$, that is, $18·50. Now since 50 cents is just half of one dollar they must have been worth half of $18·50=$9·25.

In order to work by this rule, we must make use of *aliquot parts.* An aliquot part of any thing is an exact part. In the above example, 50 cents is an aliquot part of $1, since it is exactly half of $1. We will give some aliquot parts which are in frequent use, in the following

TABLE OF ALIQUOT PARTS.

cts.	$	*mo.*	*yr.*	*s.*	£	*d.*	*s.*
50 =	$\frac{1}{2}$	6 =	$\frac{1}{2}$	10 =	$\frac{1}{2}$	6 =	$\frac{1}{2}$
$33\frac{1}{3}$ =	$\frac{1}{3}$	4 =	$\frac{1}{3}$	6 8*d.* =	$\frac{1}{3}$	4 =	$\frac{1}{3}$
25 =	$\frac{1}{4}$	3 =	$\frac{1}{4}$	5 =	$\frac{1}{4}$	3 =	$\frac{1}{4}$
20 =	$\frac{1}{5}$	2 =	$\frac{1}{6}$	4 =	$\frac{1}{5}$	2 =	$\frac{1}{6}$
$16\frac{2}{3}$ =	$\frac{1}{6}$	1 =	$\frac{1}{12}$	3 4*d.* =	$\frac{1}{6}$	$1\frac{1}{2}$ =	$\frac{1}{8}$
$12\frac{1}{2}$ =	$\frac{1}{8}$	15*da.* =	$\frac{1}{2}$ of 1*mo.*	2 6*d.* =	$\frac{1}{8}$	$1\frac{1}{3}$ =	$\frac{1}{9}$
10 =	$\frac{1}{10}$	10 =	$\frac{1}{3}$ "	2 =	$\frac{1}{10}$	$1\frac{1}{5}$ =	$\frac{1}{10}$
$8\frac{1}{3}$ =	$\frac{1}{12}$	6 =	$\frac{1}{5}$ "	1 8*d.* =	$\frac{1}{12}$	1 =	$\frac{1}{12}$ *d.*
$6\frac{1}{4}$ =	$\frac{1}{16}$	5 =	$\frac{1}{6}$ "	1 4*d.* =	$\frac{1}{15}$	2*far.* =	$\frac{1}{2}$ of 1.
5 =	$\frac{1}{20}$	3 =	$\frac{1}{10}$ "	1 =	$\frac{1}{20}$	1 =	$\frac{1}{4}$ "

What is Practice? What is an aliquot part of any thing? Repeat all the aliquot *parts of a dollar as given in the above table.* Repeat in the same way all the other *aliquot parts of the table.*

EXAMPLES.

1. What will 435 yards of cloth cost, at $0·75 per yard?

435 yards, at $1 per yard=$435.
435 yards, at 50 cents per yard=½ of $435=217·50.
435 yards, at 25 cents per yard=¼ of $435=108·75.
435 yards, at 75 cents per yard= $326·25.

2. What cost 13½ pounds of tea, at 5*s.* 6*d.* per pound?

13*lb.* at 5*s.* =65*s.*=£3 5*s.*
13*lb.* at 6d. or ½*s.*= 6 6*d.*
½*lb.* at 5*s.* = 2 6
½*lb.* at 6*d.* = 3
£3 14*s.* 3*d.*

3. What cost 37½ dozen of eggs, at 1*s.* 4*d.* per dozen?

37*doz.* at 1*s.* per *doz.*=37*s.*=£1 17*s.*
37*doz.* at 4*d.* or ⅓*s.* =12⅓= 12 4*d.*
½*doz.* at 1*s.* = 6
½*doz.* at 4*d.* = 2
Ans. £2 10*s.*

4. If I receive 7 dollars for the use of 100 dollars for one year, how much ought I to receive for the use of 100 dollars for 7 months, 18 days?

1 year or 12 months gives $7
6 months equals ½ of a year, gives $3·50
1 month equals ⅙ of 6 months, 58⅓
15 days equals ½ of 1 month, 29⅙
3 days equals ⅕ of 15 days, 5⅚
Ans. $4·43⅓

5. What cost 7½ cords of wood, at $2·75 per cord?

Ans. $20·625.

6. What is the value of 28¾ pounds of butter, at 11 cents per pound? *Ans.* $3·1625.

7. What is the value of 500½ yards of tape, at 2¼ cents per yard? *Ans.* $11·26125.

8. What must I give for 13¾ bushels of oats, at 2*s.* 4*d.* per bushel? *Ans.* £1 12*s.* 1*d.*

9. What cost 18¾ pounds of ham at 8 cents per pound? *Ans.* $1·50.

10. What cost 15¾ gallons of oil, at $0·75 cents per gallon? *Ans.* $11·8125.

11. What cost 4000 quills, at $2·25 per 1000? *Ans.* $9.

12. What cost 27¾ yards of carpeting at 6*s.* 6*d.* per yard? *Ans.* £9 0*s.* 4½*d.*

13. What is the value of 25 bushels of potatoes, at $0·31¼ per bushel? *Ans.* $7·8125.

14. What is the value of 54 spelling-books, at 12½ cents per copy? *Ans.* $6·75.

15. What is the value of 47½ reams of paper, at $3·25 per ream? *Ans.* $154·375.

16. What is the value of 30½ gross of almanacs, at $2·25 per gross? *Ans.* $68.625.

17. What cost 16¾ gallons of vinegar, at 1*s.* 4*d.* per gallon? *Ans.* 1£ 2*s.* 4*d.*

18 What is the value of 5⅓ bushels of walnuts, at 8*s.* 6*d.* per bushel? *Ans.* £2 5*s.* 4*d.*

19. What cost 3½ gross of matches, at $1·125 per gross? *Ans.* $3·9375.

20. What cost 325 bushels of apples, at 37½ cents per bushel? *Ans.* $121·875.

21. What cost 16½ yards of cloth, at $3¾ per yard? Ans. $61·875

22. If the interest on a certain sum of money is $7·35 in one year, how much will it be for 5½ months? *Ans.* $3·36⅞.

23. If the interest of $100 for one year is $6, how much is it for 10 months and 10 days? *Ans.* $5·16⅔.

24. If a steam locomotive pass 18 miles in one hour, how far will it move in 50¼ minutes? *Ans.* 15$\frac{3}{40}$.

25. If the interest of $100 for 12 months is $7, how much is it for 4⅓ months? *Ans.* 2·52$\frac{7}{9}$.

26. What must I pay for 1¼ cords of wood, 128 feet in a cord, at 6¼ cents per foot? *Ans.* $10.00.

PERCENTAGE.

112. The term *per cent.* is an abbreviation of *per centum*, which means by the hundred.

Thus, 5 out of 100 is 5 per cent.
6 out of 100 is 6 per cent.
7 out of 100 is 7 per cent.

And so for other rates per cent.

Different rates per cent. are most conveniently expressed by means of decimals.

Thus, 1 per cent. is the same as 0·01.
2 " " 0·02.
3 " " 0·03.
4 " " 0·04
5 " " 0·05.

In many cases the rate per cent. is very concisely expressed by means of a vulgar fraction, as follows:

1	per cent.	$=\frac{1}{100}$.	10	per cent.	$=\frac{10}{100}=\frac{1}{10}$.
2	"	$=\frac{2}{100}=\frac{1}{50}$.	20	"	$=\frac{20}{100}=\frac{1}{5}$.
4	"	$=\frac{4}{100}=\frac{1}{25}$.	25	"	$=\frac{25}{100}=\frac{1}{4}$.
5	"	$=\frac{5}{100}=\frac{1}{20}$.	50	"	$=\frac{50}{100}=\frac{1}{2}$.

Suppose we wish 5 per cent. of $1122, we must take $\frac{5}{100}$ of it; this is done by multiplying by the decimal 0·05.

OPERATION.

$$\begin{array}{r} \$1122. \\ \underline{0{\cdot}05.} \\ \textit{Ans. } \$56{\cdot}10. \end{array}$$

Hence, to find the percentage of any number, we have this

RULE.

Multiply the given number by the percentage expressed in decimals, and the product will give the percentage sought.

From what is *per cent.* abbreviated? And what does it mean? 5 out of 100 is what per cent.? 6 out of 100 is what per cent.? 7 out of 100 is what per cent? What is the decimal expression for 1 per cent.? What for 2, 3, 4, &c., per cent.? Repeat the Rule for finding the per cent. of a number.

EXAMPLES.

1. What is $4\frac{1}{2}$ per cent. of $10000?

$$\begin{array}{l} 4 \text{ per cent. is } 0{\cdot}04. \\ \underline{\tfrac{1}{2} \text{ per cent. is } 0{\cdot}005.} \\ \underline{4\tfrac{1}{2} \text{ per cent. is } 0{\cdot}045.} \end{array}$$

OPERATION.

```
     $10000
      0·045
     ------
      50000
     40000
     ------
Ans. $450·000
```

2. What is 1 per cent. of $730? *Ans.* $7·30.

3. What is 3 per cent. of 5789 pounds? *Ans.* 173·67*lb.*

4. What is 4 per cent. of 365 bushels? *Ans.* 14·6*bu.*

5. What is $4\frac{1}{2}$ per cent. of $75·03? *Ans.* $3·37635.

6. What is 7 per cent. of 2345? *Ans.* 164·15.

7. What is 30 per cent. of $495? *Ans.* $148·50.

8. A person laid out $222 as follows: he gave 21 per cent. of his money for calicoes; 15 per cent. for thread; 45 per cent. for silks; and the remaining 19 per cent. for broadcloths. How many dollars did he expend for each?

Ans.

He gave for calicoes,	$46·62.
" thread,	33·30.
" silks,	99·90.
" broadcloths,	42·18.

9. A merchant having 500 barrels of flour, sold at one time 25 per cent. of it, at another time he sold 20 per cent. of the remainder. How many barrels did he sell at each time, and how many remain?

Ans.

The first time he sold	125 barrels.
The second time he sold	75 barrels.
He has remaining . .	300 barrels.

10. A farmer raising 1097 bushels of wheat, gives 10 *per cent.* of it for thrashing, 10 per cent. of the remainder

for flouring. What per cent. of the whole will he have left? *Ans.* 81 per cent.

11. A California miner having secured $15\frac{1}{2}$ pounds of gold dust, finds it to lose 5 per cent. in refining; he then gives 6 per cent. for coining. How much ought he to receive after it is coined?

Ans. 13·8415 pounds $=$ 13*lb.* 10*oz.* 1*pwt.* $23\frac{1}{25}$*gr.*

12. Suppose at each stroke of the piston of an air pump 10 per cent. of the air in the receiver is exhausted, what per cent. of the air will remain after the 1st, 2d, 3d, and 4th strokes, respectively?

Ans.
After the 1st stroke 90 per cent.
" " 2d " 81 " "
" " 3d " $72\frac{9}{10}$ " "
" " 4th " $65\frac{61}{100}$ " "

SIMPLE INTEREST.

113. Interest is money paid by the borrower to the lender, for the use of the money borrowed.

It is estimated at a certain rate *per cent. per annum*, that is, a certain number of dollars for the use of $100, for one year.

Thus, when $6 is paid for the use of $100, for one year the interest is said to be at 6 *per cent.*

In the same manner when $5 is paid for the use of $100, for one year, the interest is said to be at 5 *per cent.*; and the same for other rates.

The rate per cent. is generally fixed by law. In the

New England States the legal rate is 6 *per cent.*, while in the State of New York it is 7 *per cent.*

The sum of money borrowed, or upon which the interest is computed, is called the *principal.*

The principal, with the interest added to it, is called the *amount.*

What is Interest? How is it estimated? What is the rate per cent. when $6 is paid for the use of $100 for one year? What is the rate per cent. when $5 is in the same way paid? Is the rate per cent. generally fixed by law? What is the legal rate per cent. in the New England States? What is it in the State of New York? What is the principal? What is the amount?

CASE I.

To find the interest on any given principal, for any whole number of years, at any given rate per cent.

Suppose we wish the interest of $365·50 for 3 years, at 7 per cent.

By the definition of interest money, it follows that the interest of $365·50 for one year, at 7 per cent., is 7 per cent. of $365·50; which by rule under ART. **112**, we obtain, by multiplying $365·50 by 0·07. Performing the multiplication, we have $\$365·50 \times 0·07 = \$25·585$ for one year's interest, which, multiplied by 3, the number of years, will give us $76·755 for the interest of $365·50 for 3 years at 7 per cent.

Hence the following

RULE.

Multiply the principal by the rate per cent., expressed in decimals, and that product by the number of years; observing the usual rule for pointing off the decimal figures.

EXAMPLES.

1. What is the interest of $573·15 for 5 years at 6 per cent.?

OPERATION.

$573·15=the principal.
0·06=rate per cent.
$34·3890=one year's interest.
5=number of years.
Ans. $171·945=five years' interest.

2. What is the interest of $600 for 4 years at 5 per cent.? *Ans.* $120.

3. What is the interest of $725 for 8 years at 4½ per cent.? *Ans.* $261.

4. What is the interest of $149 for 5 years at 2 per cent.? *Ans.* $14·90.

5. What is the interest of $225·25 for 10 years at 4 per cent.? *Ans.* $90·10.

6. What is the interest of $311·30 for 11 years at 10 per cent.? *Ans.* $342·43.

7. What is the interest of $501·50 for 2 years at 3½ per cent.? *Ans.* $35·105.

CASE II.

To find the interest on any given principal, for any given time, at any given rate per cent.

Suppose we wish the interest of $126 for 3 years 5 months and 15 days, at 7 per cent.

OPERATION.

$126

0·07

$8·82 = 1 year's interest.

3

$26·46 = 3 years' "

4 mos. = $\frac{1}{3}$ of a yr. 2·94 = 4 months' "

1 mo. = $\frac{1}{4}$ of 4 mos. 735 = 1 " "

15 dys. = $\frac{1}{2}$ of 1 mo. 3675 = 15 days' "

Ans. $30·5025 = 3 yrs. 5 mos. and 15 days' int.

Hence the following

RULE.

Multiply the principal by the rate per cent. expressed in decimals; the product will give one year's interest, which, multiplied by the number of years, will give the interest for the time expressed in years. Then find the interest for the months and days by means of aliquot parts, as in Practice.

EXAMPLES.

1. What is the interest of $39·42 for 1 year, 5 months, and 11 days, at 7 per cent.?

OPERATION.

$39·42

0·07

2·7594 = 1 year's interest.

4 months = $\frac{1}{3}$ of a year 920 = 4 months' "

1 month = $\frac{1}{4}$ of 4 months 230 = 1 month's "

10 days = $\frac{1}{3}$ of 1 month ·77 = 10 days' "

1 day = $\frac{1}{10}$ of 10 days 8 = 1 day's "

$3·994 Ans.

Note.—We have not extended our work to more than three places of decimals, but have added 1 to the third place whenever the fourth decimal figure would be 5 or greater.

2. What is the interest of $47·13 for 7 months and 21 days, at 7 per cent.? *Ans.* 2·118.

3. What is the interest of $321·21 for 3 months and 15 days, at 6 per cent.? *Ans.* $5·621.

4. What is the interest of $270 for 2 months and 8 days, at 7 per cent.? *Ans.* $3·57.

5. What is the interest of $404·44 for 1 year, 5 months and 4 days, at 7 per cent.? *Ans.* $40·422.

6. What is the interest of $99·99 for 11 months and 29 days, at 5 per cent.? *Ans.* $4·986.

7. What is the interest of $37·50 for 6 months and 10 days, at 6¼ per cent.? *Ans.* $1·287.

8. What is the interest of $49·49 for 8 months and 8 days, at 7 per cent.? *Ans.* $2·386.

CASE III.

To find the interest on any given principal for any given time at 6 per cent.?

The interest on $100 for one year, at 6 per cent., being $6, it follows that the interest on $1, for one year, is $0·06; and since 2 months is $\frac{2}{12}=\frac{1}{6}$ of a year, the interest on $1, for two months, is $0·01; again, since 6 days is $\frac{6}{60}=\frac{1}{10}$ of 2 months, when we reckon 30 days to each month, it follows that the interest on $1, for 6 days, is $0·001. Hence, *if we call half the number of months,* CENTS, *and one-sixth the number of days,* MILLS, *we shall obtain the interest of* $1 *for the given time, at* 6 *per cent.*

The interest of $1 being multiplied by the number of *dollars in the given* principal, will obviously give the in terest sought. As an example, suppose we wish the interest of $125 for 1 year, 5 months and 18 days, at 6 per cent.

$0·085 = int. of $1 for 1 y. 5 m. = 17 months.
3 = " " " 18 days.
$0·088 = int. of $1 for 1 y. 5 m. and 18 days.

If now we multiply $0·088 by 125, the number of dollars in the principal; or, which is the same thing, if we multiply $125 by 0·088, we shall find $125 × 0·088 = $11, for the interest sought.

Hence we have this

RULE.

I. Call half the number of months, CENTS; *one-sixth the number of days,* MILLS; *and the result will be the interest of* $1 *for the given time.*

II. Multiply the interest of $1, *thus found, by the number of dollars in the given principal, and the product being pointed off by the rule for decimals, will give the interest required.*

EXAMPLES.

1. What is the interest of $49·37, for 13 months and 15 days, at 6 per cent.?

In this example, we find the interest on $1, for 13 months and 15 days, at 6 per cent., to be $0·0675, which, multiplied by 49·37, the number of dollars in the principal, gives $3·332475, for the interest on $49·37, for the given time.

2. What is the interest of $608·62, for 1 year and 9 months, at 6 per cent.? *Ans.* $63·9051.

3. What is the interest of $341·13, for 7 years and 9 *days, at 6* per cent.? Ans. $143·786295.

4. What is the interest of $100, for 16 years and 8 months, at 6 per cent.? *Ans.* $100.

5. What is the interest of $591·03, for 4 years, 3 months and 7 days, at 6 per cent.? *Ans.* $151·402185.

6. What is the interest of $0·134, for 4 months and 3 days, at 6 per cent.? *Ans.* $0·002747.

7. What is the interest of $7·50, for 7 months, at 6 per cent.? *Ans.* $0·2625.

8. What is the interest of $371·01, for 4 years and 15 days, at 6 per cent.? *Ans.* $89·969925.

9. What is the interest of $57·92, for 3 years, 7 months and 9 days, at 6 per cent.? *Ans.* $12·53968.

10. What is the interest of $329, for 5 years and 13 days, at 6 per cent.? *Ans.* $99·412⅚.

11. What is the interest of $47·39, for 1 year and 7 months, at 6 per cent.? *Ans.* $4·50205.

CASE IV.

To find the interest on any given principal, for any given time, at any given rate per cent.

Interest at 6 per cent. increased by ⅙ of itself will obviously give the interest at 7 per cent. The interest at 6 per cent. increased by ⅓ of itself will give the interest at 8 per cent. If we diminish the interest at 6 per cent. by ⅙ of itself, we shall obtain the interest at 5 per cent. And in all cases, by increasing or decreasing the interest at 6 per cent., in the proper ratio, we may obtain the interest at any other desired rate.

As an example, suppose we wish the interest of $300 for 1 year, 3 months, and 12 days, at 4½ per cent.

By Case III. we readily find the interest of $300 for 1 year, 3 months and 12 days, at 6 per cent., to be $23·10.

But the interest is required at 4½ per cent. instead of at 6 per cent. If ¼ of 6 be taken from 6, the remainder will be 4½; hence, if ¼ of $23·10, the interest at 6 per cent., be taken from $23·10, the remainder will be the interest at 4½ per cent. Performing this operation, we have $23·10 −¼ of $23·10=$17.325 for the interest of $300 for 1 year, 3 months and 12 days, at 4½ per cent.

Hence, we have this

RULE.

Find the interest on the given principal, for the given time, at 6 *per cent., by Case III. Then increase, or decrease, this interest by the same part of itself, as it would be necessary to increase, or decrease* 6 *per cent., in order to make it agree with the given rate per cent.*

EXAMPLES.

1. What is the interest of $19·41, for 1 year, 7 months and 13 days, at 7 per cent.?

In this example, we find by Case III. that the interest of $19·41, for 1 year, 7 months and 13 days, at 6 per cent., is $1·886005. Since 6, increased by its sixth part, equals 7, it will be necessary to increase the interest just found for 6 per cent., by its sixth part, which becomes $2·200339⅙, for the interest at 7 per cent.

2. What is the interest of $530, for 3 years and 6 months, at 5 per cent.? *Ans.* $92·75.

In this example, it was necessary to decrease the interest of 6 per cent., by its sixth part.

3. What is the interest of $5·37, for 4 years and 12 days, at 8 per cent.? *Ans.* $1·73272.

In this example, we increase the interest at 6 per cent. by its third part.

4. What is the interest of $4070, for 3 months, at 9 per cent.? *Ans.* $91·575.

5. What is the interest of $3671, for 6 months, at 10 per cent.? *Ans.* $183·55.

6. What is the interest of $4920·05, for 3 months, at 4 per cent.? *Ans.* $49·2005.

7. What is the interest of $40·17, for 3 months and 18 days, at 3 per cent.? *Ans.* $0·36153.

8. What is the interest of $37·13, for 5 months and 12 days, at $4\frac{1}{2}$ per cent.? *Ans.* $0·7518825.

9. What is the interest of $489, for 3 years and 4 months, at $5\frac{1}{2}$ per cent.? *Ans.* $89·65.

10. What is the interest of $700, for 1 year and 9 months, at 7 per cent.? *Ans.* $85·75.

NOTE.—When the principal is given in English money, we must reduce the shillings, pence and farthings, to the decimal of a £; and then proceed as in Federal money.

11. What is the interest of £75 13*s.* 6*d.*, for 3 years and 5 months, at 6 per cent.?

In this example, 13*s.* 6*d.*, reduced to the decimal of a £, is 0·675, so that our principal is £75·675; the interest on £1, for 3 years and 5 months, at 6 per cent., is £0·205, which, multiplied by 75·675, gives £15·513375=£15 10*s.* $3\frac{21}{100}$*d.*, for the interest required. (See ART. **99.**)

12. What is the interest of £14 5*s.* $3\frac{1}{2}$*d.*, for 4 years 6 months and 14 days, at 7 per cent.?

Ans. £4 10*s.* $7\frac{72}{100}$*d.* nearly.

13. What is the interest of £1 7*s.* 6*d.*, for 2 years and 6 months, at $4\frac{1}{2}$ per cent.? *Ans.* £0 3*s.* $1\frac{1}{8}$*d.*

14. What is the interest of £105 10*s.* 6*d.*, for $9\frac{1}{2}$ *months, at 5 per cent.?* Ans. £4 3s. 6d. 1·95far.

INTEREST WHEN THE TIME IS ESTIMATED IN DAYS.

114. Thus far, we have considered the time, for which interest is to be computed, as estimated in months and days, counting a month as $\frac{1}{12}$ of a year, and 1 day as $\frac{1}{30}$ of a month, or $\frac{1}{360}$ of a year.

Now, as some months have 31 days, and others less than 31, we, by the previous methods, obtain sometimes too much interest, and sometimes too little, but the error must always be small.

We will, under this Article, explain the more accurate method by means of days, which is sometimes called the *Commercial Method.*

Suppose we wish the interest of $500 from May 15th to November 20th, at 7 per cent.

By Case I., Art. **113**, we find $\$500 \times 0{\cdot}07 = \$35$ for one year's interest of $500, at 7 per cent. By Table under Art. **76**, we find 189 days from May 15th to November 20th.

It is obvious that the interest for 189 days must be the same fractional part of one year's interest, that 189 days is of 365 days. Hence, $\$35 \times \frac{189}{365} = \frac{\$35 \times 189}{365} = \$18{\cdot}123+$ for the interest of $500 from May 15th to November 20th, at 7 per cent.

Hence this

RULE.

Multiply the principal by the rate per cent. expressed in decimals; the product will be one year's interest; which multiply by the time expressed in days, and divide this last product by 365, *and the quotient will be the interest sought.*

EXAMPLES.

1. A note of $37·37 was given May 3, 1848; how much was due on it Dec. 27, 1848, at 7 per cent.?

By the table under Art. **76**, we find 238 days from May 3 to Dec. 27.

OPERATION.

$37·37=principal.
0·07=rate per cent.
$2·6159=one year's interest.
238=time in days.
209272
78477
52318
365)622·5842(1·705=interest sought in dollars
365 37·37 =principal.
2575 $39·075=amount. *Ans.*
2555
2084
1825
259

2. A note of $365 was given July 4, 1847; what will it amount to, June 1, 1849, interest being 7 per cent.? *Ans.* $413.79.

3. What is the interest on $100 from January 13th to November 15, it being Leap-year, and interest being 6 per cent.? *Ans.* $5·047.

4. What is the interest on $216 from March 10th to December 1st, interest being 5 per cent.? *Ans.* $7·871.

5. What is the interest on $107 from April 12th to July 4th, interest being 7 per cent.? *Ans.* $1·703.

6. What is the interest on $1000 from June 20th to August 13th, interest being 7 per cent.? *Ans.* $10·356.

7. What is the interest on $730 from July 4th to December 25th, interest being 6 per cent.? *Ans.* $20·88.

8. What is the interest on $63·37 from August 9th to December 31st, interest being 7 per cent.? *Ans.* $1·75.

PARTIAL PAYMENTS.

115. When notes, bonds, or obligations receive partial payments, or indorsements,* the rule adopted by the Supreme Court of the United States is as follows:

RULE.

"*The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceed the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments taken together exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance, as aforesaid.*"

The above rule has been adopted by *New York*, *Massa-*

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* From a Latin phrase, *in dorso*, meaning "upon the back;" because the payments are written across the back of the note.

chusetts, and by nearly all the other states of the Union. The following is the

CONNECTICUT RULE.

"*Compute the interest on the principal to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner, from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more. But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.*

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period"

EXAMPLES.

$620. UTICA, NOV. 1, 1837.

1. For value received, I promise to pay Thomas Jones, or order, the sum of six hundred and twenty dollars, on demand, with interest. CHARLES BANK.

The following indorsements were made on this note:

1838, Oct. 6, there was indorsed . . . $61·07.
1839, March 4, " " " . . . . . 89·03.
1839, Dec. 11, " " " . . . . . 107·77.
1840, July 20, " " " . . 200·50.

What was the balance due, Oct. 15, 1840, allowing 7 per cent. interest, according to the U. S. rule?

The pupil will find it convenient to arrange the work for finding the *multipliers* at 6 per cent. as follows:

	year.	*mo.*	*da.*	*mo.*	*da.*	*multipliers at* 6 *p. c*
Date of note	1837	10	1			
1st indorsement	1838	9	6	11	5	0·055⅚.
2d indorsement	1839	2	4	4	28	0·024⅔.
3d indorsement	1839	11	11	9	7	0·046⅙.
4th indorsement	1840	6	20	7	9	0·0365.
Date of settlement	1840	9	15	2	25	0·014⅙.
				35	14	0·177⅓.

Having found the different intervals of time, we then find the multipliers at 6 per cent. by CASE III. ART. **113.**

As a check upon our work, we add all the different intervals of time together, and find it makes 35 months and 14 days. We also add all the multipliers, and obtain 0·177⅓.

Now, subtracting the time the note was given from the time of settlement, we also obtain 35 months and 14 days, which time gives 0·177⅓ for multiplier.

It would be well in all cases where interest is to be cast on a note of many indorsements, to follow the above method of operation; since, by proceeding with systematic order, there is less chance for committing errors.

If we wish to have the result true to a cent, we must carry our work to three decimal places, or to mills.

In the following operation we extend the work to three decimal places; when the value beyond the third place is one half or more, we add a unit to the decimal in the third place, but when that value is less than one half, we *neglect it.*

Having found the *multipliers*, we continue the work as follows:

The amount of note, or principal, is	$620·000
Interest on the same, up to Oct. 6, 1838, is	40·386
Amount due on note, Oct. 6, 1838, is	660·386
The first indorsement is	61·070
	599·316
Interest from Oct. 6, 1838, to March 4, 1839, is	17·247
Amount due March 4, 1839, is	616·563
The second indorsement is	89·030
	527·533
Interest from March 4, 1839, to Dec. 11, 1839, is	28·414
	555·947
The third indorsement is	107·770
	448·177
Interest from Dec. 11, 1839, to July 20, 1840, is	19·085
	467·262
The fourth indorsement is	200·500
	266·762
Interest from July 20, 1840, to Oct. 15, 1840, is	4·409
Ans.	271·171

$350. UTICA, May 1, 1836.

2. For value received, I promise to pay Isaac Clark, or order, three hundred and fifty dollars, with interest, at 6 per cent. N. BROWN.

Indorsements were made on this note as follows:

Dec. 25, 1836, there was paid	$50.
June 30, 1837, " " "	5.
Aug. 22, 1838, " " "	15.
June 4, 1839, " " "	100.

How much was due April 5, 1840, if interest is computed according to the U. S. rule?

	year.	*mo.*	*da.*		*mo.*	*da.*	*multipliers at 6 per cent.*
Date of note	1836	4	1				
1st indorsement	1836	11	25		7	24	0 039.
2d indorsement	1837	5	30	*yr.*	6	5	0·030$\frac{5}{6}$.
3d indorsement	1838	7	22	1	1	22	0·068$\frac{2}{3}$.
4th indorsement	1839	5	4		9	12	0·047.
Date of settlement	1840	3	5		10	1	0·050$\frac{1}{6}$.
				3	11	4	0·235$\frac{2}{3}$.

The amount of the note, or principal, is . .		$350·000
Interest up to Dec. 25, 1836, is		13·650
		363·650
The first indorsement is		50·000
		313·650
Interest up to June 4, 1839, is		45·950
		359·600
Indorsement, June 30, 1837, which is less than the interest then due,	$5	
Indorsement, Aug. 22, 1838, . . .	15	
This sum is still less than the interest now due, ... ,	20	
Indorsement, June 4, 1839, ...	100	
		$120·000
This sum exceeds the interest now due.		
		239·600
Interest up to April 5, 1840, is		12·020
Amount due April 5, 1840, ... *Ans.*		$251·620

How much would have been due, had we computed *interest* according to the Connecticut Rule?

NOTE.—We will, here, indicate the steps of the process under the Connecticut Rule. First, find the amount of the principal sum for one year; that is, to May 1, 1837. Then find the amount of the first payment to the same date. Deduct the latter amount from the former. Next, find the amount of the new principal thus obtained for another year, that is, to May 1, 1838; then find the amount of the second payment to the same time, and deduct as before, and so on. *Ans.* \$252·12.

\$$108\frac{43}{100}$. UTICA, Dec. 9, 1835.

3. For value received, I promise to pay Peter Smith, or order, one hundred and eight dollars and forty-three cents, on demand, with interest, at 7 per cent.

JOHN SAVEALL.

Indorsements were made as follows:

March 3, 1836, there was indorsed . \$50·04.
Dec. 10, 1836, " " " . . . 13·19.
May 1, 1838, " " " . . . . 50·11.

How much remained due, according to the U. S. rule, Oct. 9, 1840? *Ans.* \$5·844.

How much according to the Connecticut Rule?

NOTE.—After several steps, there will be a new principal, Dec. 9, 1838. The interest is to be computed upon this, not for one year, since there is no indorsement within the year, but up to the time of settlement. *Ans.* \$5·80.

\$$143\frac{50}{100}$. UTICA, August 1, 1837.

4. For value received, I promise to pay D. Budlong, or bearer, one hundred and forty-three dollars and fifty cents, on demand, with interest. W. GOULD.

Dec. 17, 1837, there was indorsed . . \$37.40.
July 1, 1838, " " " . . . . . 7·09.
Dec. 22, 1839, " " " . . . 13·13.
Sept. 9, 1840, " " " . . . . . 50·50.

How much remains due, according to each rule, Dec. 28, 1840, the interest being 7 per cent.?

Ans. U. S. rule, $60.866.

NOTE.—After a few steps we shall find a new principal, Aug. 1, 1838. We compute the interest on this up to Dec. 22, 1839, as there is no payment within a year. From the amount deduct the payment made Dec. 22, 1839. We have, then, another new principal, the interest on which is to be computed for one year, that is, to Dec. 22, 1840, and added; we find also the amount of the last payment to that date; deduct, and find amount of the balance, Dec. 28, 1840. *Ans.* Conn. rule, $60·72.

5. A note of $486 is dated Sept. 7, 1831, on which,

March	22, 1832,	there was paid . . .	$125.
Nov.	29, 1832,	" " "	150.
May	13, 1833,	" " "	120.

What was the balance due, according to each rule, April 19, 1834, the interest being 7 per cent.?

Ans. { U. S. rule $144·404.
Conn. " 143·55.

116. The principal, the rate per cent., the time and the interest, are so related to each other, that any three of them being given, the remaining one can be found.

PROBLEM I.

Given the principal, the rate per cent., and the time, to find the interest. The rule for this problem has already been given under Case III., ART. **113**; it is equivalent to *the following*

RULE.

Multiply the interest of $1, for the given time and given rate per cent., by the number of dollars in the principal.

PROBLEM II.

Given the time, the rate per cent., and the interest, to find the principal. It is obvious that the interest on a given sum is as many times greater than the interest on $1, as the given sum is times greater than $1. Hence, we have the following

RULE.

Divide the given interest by the interest of $1, for the given time, and given rate per cent.

EXAMPLES.

1. The interest on a certain principal, for 9 months and 10 days, at 4½ per cent., is $1·01605. What was the principal?

In this example, we find the interest of $1, for 9 months and 10 days, at 4½ per cent., to be $0·035; therefore, $1·01605, divided by $0·035, gives 29·03, for the number of dollars in the principal required.

2. What principal will, in 1 year, 7 months, and 15 days, at 6 per cent., give $9·75 interest? *Ans.* $100.

3. What principal will, in 7 years and 9 days, at 6 per cent., give $16·86 interest? *Ans.* $40.

4. What principal will, in 3 years and 6 months, at 5 *per cent.*, give $92·75 interest? *Ans.* $530.

5. What principal will, in 3 months and 9 days, at 8 per cent., give $90 interest? *Ans.* $4090·909

PROBLEM III.

Given the principal, the time, and the interest, to find the rate per cent.

If the rate per cent. be doubled, other things being the same, the interest will be doubled; if the rate per cent. is tripled, the interest will be tripled. And, in all cases, the interest at any particular rate per cent. is as many times greater than the interest at 1 per cent. as the given rate per cent. is times greater than 1 per cent. Hence, we have this

RULE.

Divide the given interest by the interest of the given principal, for the given time, at 1 *per cent.*

EXAMPLES.

1. The interest of $100, for 9 months and 10 days, is $3 50. What is the rate per cent.?

In this example, we find the interest of $100, for 9 months and 10 days, at 6 per cent., to be $4 66⅔. The interest at 1 per cent. is $0·77 7⁄9; therefore, dividing $3·50 by $0·77 7⁄9, we obtain 4½ for the rate per cent. required.

2. At what rate per cent. will $530, in 3 years and 6 months, give $92·75 interest? *Ans.* 5 per cent.

3. At what rate per cent. will $19·41, in 1 year, 7 months, and 13 days, give $2·200339⅙ interest?

Ans. 7 per cent.

4. At what rate per cent. will $5·37, in 4 years and 12 days, give $1·73272 interest? *Ans.* 8 per cent.

5. At what rate per cent. will $4070, in 3 months, give $91·575 interest? *Ans.* 9 per cent.

PROBLEM IV.

Given the principal, the rate per cent., and the interest, to find the time.

If the time for which interest is computed be doubled, other things being the same, the interest will be doubled; if the time is tripled, the interest will be tripled. And in all cases, the interest for any particular time is as many times greater than the interest for one year, as the particular time is greater than 1 year. Hence, we have this

RULE.

Divide the given interest by the interest of the given principal, for 1 *year, at the given rate per cent.*

EXAMPLES.

1. In what time will $37·13, at 4½ per cent., give $0·7518825 interest?

In this example, we find the interest of $37·13, for 1 year, at 4½ per cent., to be $1·67085; therefore, dividing $0·7518825 by $1·67085, we get 0·45 years; this reduced to months and days, gives 5 months and 12 days.

2. In what time will $700, at 7 per cent., give $85·75 interest? *Ans.* 1 year, 9 months.

3. In what time will $100, at 6 per cent., give $100 interest? That is, in what time will a given principal double itself at 6 per cent. interest? Ans. 16⅔ years.

4. In what time will a given principal double itself at 7 per cent.? *Ans.* $14\frac{2}{7}$ years.

5. In what time will a given principal double itself at 8 per cent.? *Ans.* $12\frac{1}{2}$ years.

6. In what time will a given principal double itself at 5 per cent.? *Ans.* 20 years.

7. In what time will a given principal double itself at $4\frac{1}{2}$ per cent.? *Ans.* $22\frac{2}{9}$ years.

The following table gives the time required for a given principal to double itself at simple interest at various rates per cent.

Per cent.	Years.	Per cent.	Years.	Per cent.	Years.
1	100	4	25	7	$14\frac{2}{7}$
$1\frac{1}{2}$	$66\frac{2}{3}$	$4\frac{1}{2}$	$22\frac{2}{9}$	$7\frac{1}{2}$	$13\frac{1}{3}$
2	50	5	20	8	$12\frac{1}{2}$
$2\frac{1}{2}$	40	$5\frac{1}{2}$	$18\frac{2}{11}$	$8\frac{1}{2}$	$11\frac{13}{17}$
3	$33\frac{1}{3}$	6	$16\frac{2}{3}$	9	$11\frac{1}{9}$
$3\frac{1}{2}$	$28\frac{4}{7}$	$6\frac{1}{2}$	$15\frac{5}{13}$	$9\frac{1}{2}$	$10\frac{10}{19}$

DISCOUNT.

117. DISCOUNT is an allowance made for the payment of money before it is due.

The *present worth* of a debt payable at some future time, without interest, is such a sum of money as will, if put at interest for the given time, amount to the debt.

When the interest is at 6 per cent., the amount of $1, for one year, is $1·06; therefore, the present worth of $1·06, due one year hence, is $1. If the present worth of $1·06 is $1, it follows that the present value of $1 will be the same fractional part of $1, that $1 is of $1·06;

that is, the present value of $1 is $\frac{1}{1.06}$ of $1, or $\frac{\$1}{1.06}$. And since the present worth of two dollars is twice as great as for one dollar, we have $\frac{\$2}{1.06}$ for the present worth of $2. And in the same way we find $\frac{\$3}{1.06}$ for the present worth of $3; $\frac{\$10}{1.06}$ for the present worth of $10.

Had the time been 6 months instead of one year, then would $\frac{\$1}{1.03}$ be the present worth of $1; $\frac{\$2}{1.03}$ would be the present worth of $2; $\frac{\$10}{1.03}$ would be the present worth of $10.

If we reckon 7 per cent. interest, the present worth of $1 for one year would be $\frac{\$1}{1.07}$, for 6 months it would be $\frac{\$1}{1.035}$; and so on for other sums and other rates of interest. Hence, we have the following

RULE.

Find the amount of $1, for the given time, at the given rate per cent., then divide the sum whose present worth is required, by this amount, and it will give the number of dollars in the present worth. Subtract the present worth from the given sum, and it will give the discount.

What is Discount? What is the present worth of a sum of money due at some future period? What is the present worth of $1·06, due one year hence, at 6 per cent. interest? Repeat the Rule for computing discount.

EXAMPLES.

1. What is the present worth of $622·75, due 3 years and 6 months hence, at 5 per cent.?

In this example, we find the amount of $1, for 3 years and 6 months, at 5 per cent., to be $1·175; therefore, dividing $622·75 by $1·175, we get 530, for the number of dollars in the present worth. If we subtract the present worth from the sum, we get $92·75 for the discount.

2. What is the present worth of $4161·575, due 3 months hence, at 9 per cent.? *Ans.* $4070.

3. What is the present worth of $7.10272, due 4 years and 12 days hence, at 8 per cent.? *Ans.* $5·37.

4. Sold goods for $1500, to be paid one half in 6 months, and the other half in 9 months. What is the present worth of the goods, interest being at 7 per cent.? *Ans.* $1437·227.

5. Sold goods for $1500, to be paid at the end of 7½ months. What is the present worth of the goods, interest being at 7 per cent.? *Ans.* $1437·126.

6. What is the present worth of $50, payable at the end of 3 months, at 7 per cent.? *Ans.* $49·14.

7. What is the discount on $100, due 6 months hence, at 6 per cent.? *Ans.* $2·913.

8. What is the discount on $750, due 9 months hence, at 7 per cent.? *Ans.* $37·411.

9. What is the present worth of $3471·20, due 3 years and 9 months hence, at 4½ per cent.? *Ans.* $2970·011.

10. What is the discount of $150, due 3 months and 18 days hence, at 6 per cent.? *Ans.* $2·652.

11. What is the discount of $961·13, due 1 year and 5 months hence, at 7 per cent.? *Ans.* $86·713.

12. What is the discount of $37·40, due at the end of 7 months, at 6 per cent.? *Ans.* $1·265.

13. Bought a bill of goods for $1200, one third payable in 3 months, one third in 6 months, and the remaining one third in 9 months. How much ready cash ought to pay for the goods, if we consider money with 6 per cent.? *Ans.* $1165·21+.

COMPOUND INTEREST.

118. When, at the end of a year, or of any given time, the interest due is added to the principal, and the amount thus obtained is considered as a new principal; upon which interest is to be cast for another given period, to be added in like manner to form a second new principal, and so on, the last amount thus obtained is called the AMOUNT AT COMPOUND INTEREST. If from this amount we subtract the original principal, we obtain the COMPOUND INTEREST.

How is the amount of compound interest found? How is the compound interest obtained?

EXAMPLES.

1. What is the compound interest of $1000, for 3 years at 7 per cent.?

Principal, ...	$1000
Interest on $1000 for one year, . .	70
First amount, or second principal, ·	1070
Interest on $1070 for one year, ...	74·90
Second amount, or third principal, .	1144·90
Interest on $1144·90 for one year, . .	80·143
Third amount, ...	1225·043
Original principal, ...	1000
The compound interest required, *Ans.*	$225·043

2. What is the amount of $100, at 6 per cent. per annum, compound interest, for two years, when the interest is added in at the end of every six months?

Principal, ...	$100
Interest on $100 for 6 months, . .	3
	103
Interest on $103 for 6 months, . .	3 09
	106·09
Interest on $106·09 for 6 months,	3·1827
	109·2727
Interest on $109·2727 for 6 months,	3·278181
Amount required, ...	$112·550881

3. What is the compound interest of $630, for 4 years, at 5 per cent.? *Ans.* $135·769.

4. What is the amount at compound interest, of $50, for 3 years, at 5 per cent.? *Ans.* $57·881.

5. What is the compound interest of $1000, for 4 years, at 6 per cent.? *Ans.* $262·477.

BANKING.

119. A BANK is an incorporated institution, created for the purpose of loaning money, receiving deposits, and dealing in exchange.

The *stock*, or amount of money in trade, is limited by law, and owned by various individuals who are called *stockholders*.

Banks are allowed to make notes which are denominated *bank bills*, which circulate as money, because, on demand of the holders of them, they must be redeemed by the banks, with *specie*.

It is customary for banks, in most cases, when they loan money, to take the interest in advance;* that is, to deduct it from the face of the note at the time the money is lent. The note is then said to be *discounted*.

The sum to be discounted, or the face of the note, is called the *amount*.

The interest deducted is called the *discount*.

What then remains is called the *present worth*, or *proceeds*.

A note to be discounted, or bankable, must be made payable at some future time, and to the order of some person who indorses it.

It is usual for the banks to take the interest for 3 days more than the time specified in the note; and the borrower is not obliged to make payment till these three days have expired, which are for this reason called *days of grace*.

What is a bank? What is the stock? Who are the stockholders? How are bank notes called? Do they circulate as money? How are the banks obliged to redeem their notes? How do banks sometimes take the interest? When is a note said to be discounted? What is the amount? What is the interest deducted called? How is that which remains called? Does a bank note require an indorser? For how many days more than specified in the note do banks take interest? What are these three days called?

What is the banking discount on $1000, for 3 months, at 7 per cent.?

In this example, we find the interest on $1, for 3 months and 3 days, at 6 per cent., to be $0·0155, which, multiplied by 1000, gives $15·50, for the discount at 6 per cent.; this, increased by its sixth part, becomes \$18·08½ for the discount at 7 per cent., as required.

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* This method of discounting bank notes is usurious, and is fast going out of use, and instead of it the banks now deduct the discount as found by Rule under ART 117.

Hence we may find banking discount by the following

RULE.

Compute the interest (*Case IV.* ART. **113**,) *on the given sum, for three days more than is specified. This interest will be the discount.*

EXAMPLES.

1. What is the banking discount of \$150, for 6 months, at 6 per cent.? *Ans.* \$4·575.

2. What is the banking discount of \$375, for 3 months and 9 days, at 7 per cent.? *Ans.* \$7·438.

3. What is the banking discount of \$400, for 9 months, at 7 per cent.? *Ans.* \$21·23⅓.

4. What is the banking discount of \$29·30, for 7 months, at 5 per cent.? *Ans.* \$0·867.

5. What is the banking discount of \$472, for 10 months, at 7 per cent.? *Ans.* \$27·809.

120. When the present worth of a bankable note, the time for which it is to be discounted, and the rate per cent. are given, to find the amount or face of the note.

What must be the face of a bank note which, when discounted for 4 months and 15 days, gives a present worth of \$100, interest being 6 per cent.?

If we suppose the note to be \$1, we find the banking discount for 4 months and 15 days to be \$0.023; hence \$1—\$0·023=\$0·977, is the present worth. Had the face of the note been \$2, the present worth would have been twice \$0.977; had it been \$10, ten times \$0·977, and *the same* would be true for other ratios. Hence, in order that the present worth of the note may be $100, its face must be as many times greater than $1, as $100 is times greater than $0·977. Dividing $100 by $0·977, we find for a quotient 102·354+. Hence, if a bank note for $102·35 be discounted for 4 months and 15 days at 6 per cent. interest, the present worth will be $100.

Hence we have this

RULE.

Compute the banking discount on $1, for the given time and rate per cent.; subtract this discount from $1, then divide the present worth by the remainder, and the quotient will be the amount, or face of the note.

EXAMPLES.

1. What must be the amount of a bankable note, so that when discounted for 3 months, at 6 per cent., it shall give a present worth of $600?

In this example, we find the banking discount on $1, for 3 months, to be $0·0155, which, subtracted from $1, gives $0·9845; therefore, dividing $600 by $0·9845, we obtain $609·446, for the required amount of the note.

2. What must be the face of a bankable note, so that when discounted for 2 months, at 7 per cent., the borrower shall receive $50? *Ans.* $50·62.

The following table gives the amount of a bankable note, so that when discounted at 5, 6, or 7 per cent., for any number of months, from 1 to 12, the present worth shall be just $1.

Months.	5 per cent.	6 per cent.	7 per cent.
1	1·004604	1·005530	1·006458
2	1·008827	1·010611	1·012402
3	1·013085	1·015744	1·018416
4	1·017380	1·020929	1·024503
5	1·021711	1·026167	1·030662
6	1·026079	1·031460	1·036896
7	1·030485	1·036807	1·043206
8	1·034929	1·042095	1·049593
9	1·039411	1·047669	1·056059
10	1·043932	1·053186	1·062605
11	1·048493	1·058761	1·069233
12	1·053093	1·064396	1·075944

We will now work some examples by the aid of the above table.

3. What must be the face of a bankable note, so that when discounted for 10 months at 5 per cent., the present worth may be $1000?

Looking in the table directly under the 5 per cent., and adjacent to 10 months, we find $1·043932; this, multiplied by 1000, gives $1043·932, for the face of the note required.

4. What must be the face of a bankable note, so that when discounted for 7 months, at 7 per cent., the present worth may be $70·50? *Ans.* $73·546.

5. What amount must I make my note, so that when discounted at the bank for 12 months, at 7 per cent., I may receive $100? *Ans.* $107·594.

6. What must be the amount of a note, so that when discounted at the bank for 6 months, at 6 per cent., the borrower may receive $365? *Ans.* $376·483.

COMMISSION.

121. Commission is an allowance made to a factor or commission merchant for buying and selling. It is estimated at so much per cent. on the money used in the transaction.

What is Commission? How is it estimated?

Since commission is a certain percentage of money employed in buying and selling goods, it may be found by the rule under *Percentage*, Art. **112**, which may be given as follows:

RULE.

Multiply the sum of money on which commission is to be computed, by the rate per cent. expressed in a decimal, and the product, when pointed off according to the rule for decimals, will be the commission.

EXAMPLES.

1. What is the commission on $3765·50, at 3½ per cent.?

OPERATION.

$$\begin{array}{r} \$3765{\cdot}50 \\ 0{\cdot}035 \\ \hline 1882750 \\ 1129650 \\ \hline \$131{\cdot}79250 \end{array}$$

2. What is the commission on $10000, at 4 per cent.?

Ans. $400.

3. A factor sells 43 bales of cotton at $375 per bale,

and charges 2 per cent. commission. How much money must he pay to his principal? *Ans.* 15802·50.

4. A sends to B, a broker, $3605 to be invested in stock: B is to receive 3 per cent. on the amount paid for the stock. What was the value of the stock purchased?

Since B is to receive 3 per cent., it is plain that $103 of A's money would purchase $100 worth of stock. Hence, the amount expended for stock must be $\frac{100}{103}$ of $3605= $3605÷1·03=$3500. *Ans.*

NOTE.—In such cases as the above, when the given sum includes the factor's commission, and we desire to know what amount he must invest for his principal, so that the balance may be his commission on the amount invested, we must divide the given sum by the percentage of the commission increased by a unit. Thus, dividing $3605 by 1·03, the quotient is $3500, which is the sum invested.

5. A factor receives $60112, and is directed to purchase cotton at $289 per bale: he is to receive 4 per cent. on the money paid for the cotton. How many bales did he purchase?

$60112÷1·04=$57800 amount paid for cotton.

$57800÷$289=200, number of bales.

6. The par value* of 125 shares of bank stock was $100 per share. What is the present value, if the stock is worth 18 per cent. above par? *Ans.* $14750.

7. What is the value of 50 shares of bank stock, the par value of which was $200 per share, on the supposition that it is 12 per cent. below par, or, that it is worth only 88 per cent of its par value? *Ans.* $8800.

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* By *par value* is meant the original cost or estimated value of stock. When it is *worth more* than its original cost, it is said to be *above par*, when it is worth less than the original cost, it is said to be *below par*.

8. A bank fails, and has in circulation $108567. It can pay only 13 per cent. What amount of money has it on hand? *Ans.* $14113·71.

INSURANCE.

122. INSURANCE is a contract, by which an individual or company agrees to restore the value of ships, houses, or goods of whatever kind, which may be destroyed by the perils of the sea, or by fire.

The security is given in consideration of a certain sum of money called the *premium*, which is paid by the owner of the property insured.

The premium is always estimated at a certain rate per cent. on the value of the property insured.

The written agreement of indemnity is called a *policy*.

What is Insurance? What is premium? How is the premium estimated? What is the policy?

It is obvious that the foregoing rules under Percentage and under Commission, may be employed for finding the insurance premium.

EXAMPLES.

1. If A gets his house insured for $1800, at 41 cents on $100, what will be the amount of the premium? *Ans.* $7·38.

2. An insurance of $12000 was effected on the ship Ocean, at a premium of 2 per cent. What did the premium amount to? *Ans.* $240.

3. I effected an insurance of $5230 on my dwelling-

house and furniture for 1 year, at ⅜ of 1 per cent. What did the premium amount to? *Ans.* $19·6125.

4. What is the amount of premium for insuring $34567, at 60 cents on $100? *Ans.* $207·402.

5. What would be the premium for insuring a ship and cargo, valued at $46370, from Boston to Liverpool, at 2¼ per cent.? *Ans.* $1043·325.

LOSS AND GAIN.

123. Loss and Gain is a rule by which merchants discover the amount lost or gained in buying and selling goods. It also assists them in adjusting the price of their goods so as to lose or gain a certain per cent.

What is Loss and Gain?

EXAMPLES.

1. Bought 300 yards of broadcloth at $2·25 per yard, and sold the same at $3·50 per yard. How much was gained?

$3·50 price of 1 yard.
$2·25 cost of 1 yard.
$1·25 gain on 1 yard.
$1·25
300
$375·00 whole gain.

2. A merchant bought 320 barrels of flour at $5 per barrel, but he finds he must lose 10 per cent. in the sales. How much will he receive for the whole?

The whole cost of 320 barrels is \$5 × 320 = \$1600.

Since he loses 10 per cent., one dollar's worth must sell for 90 cents.

$$\begin{array}{r} \$1600 \\ 0{\cdot}90 \\ \hline \end{array}$$

Ans. \$1440·00 what he receives.

3. Suppose I buy 25 cords of maple wood at \$2·50 per cord, and sell it so as to make 25 per cent. What must I receive for the whole?

The whole cost of the wood is \$2·50 × 25 = \$62·50.

Since I make 25 per cent., one dollar's worth must sell for \$1·25.

$$\begin{array}{r} \$62{\cdot}50 \\ 1{\cdot}25 \\ \hline 31250 \\ 12500 \\ 6250 \\ \hline \end{array}$$

Ans. \$78·1250 what I receive.

4. Bought a house and lot for \$1400, and sold it for \$1200. How much per cent. did I lose?

\$1400 cost of house.
\$1200 what sold for.
\$200 what I lost on \$1400.

Hence, $\frac{200}{1400} = \frac{1}{7} = 0{\cdot}14\frac{2}{7} = 14\frac{2}{7}$ per cent.

5. Bought 225 gallons of molasses for 26 cents per gallon, and sold the whole for \$64·35. What did I gain per cent.?

The whole cost of 225 gallons is \$0·26 × 225 = \$58·50. The whole gain is \$64·35 − \$58·50 = \$5·85. Since \$5·85 is the gain on \$58·50, it follows that the gain on \$1 will be found by dividing \$5·85 by 58·5. Performing the division, we have \$5·85÷58·5=0·1 or 0·10, that is, the gain is 10 per cent.

From the foregoing examples we are able to deduce the following principal

RULES.

I. The total gain or loss is the difference between the first cost and the selling price.

II. The first cost multiplied by 1, *increased by the gain per cent., or by* 1 *decreased by the loss per cent., considered as a decimal, will give the selling price.*

III. The whole gain or loss divided by the first cost, will give the gain or loss per cent.

6. Bought 75 pounds of coffee at 10 cents per pound. At how much per pound must I sell it so as to gain \$3 on the whole? *Ans.* \$0·14.

7. Bought 25 hogsheads of molasses, at \$18 per hogshead, in Havana; paid duties, \$16·30; freight, \$25; cartage, \$5·50; insurance, \$25·25. What per cent. shall I gain, if I sell it at \$28 per hogshead?

Ans. About 34 per cent.

8. If I buy broadcloth for \$3·50 per yard, how much must I sell it at per yard so as to gain 25 per cent.?

Ans. \$4·37½.

9. If I buy cloth at \$3·50 per yard, how many must I sell it at per yard so as to lose 25 per cent? *Ans.* \$2·62½.

10. A person bought a city lot for \$800, and sold it so as to gain 40 per cent. How much did he sell it for?

Ans. \$1120.

11. A house which cost \$3000 was sold for \$2400. What per cent. was lost? *Ans.* 20 per cent.

12. A house which cost \$2400 was sold for \$3000. What per cent. was gained? Ans. 25 per cent.

FELLOWSHIP.

194. Fellowship is the union of two or more individuals in trade, with an agreement to share the losses and profits in the ratio of the amount which each individual puts into the partnership. The money employed is called the *capital stock.*

The loss or gain to be shared is called the *dividend.*

What is Fellowship? What is the capital stock? What is the dividend?

EXAMPLES.

1. A, B, and C, enter into copartnership. A put in $180, B put in $240, and C put in $480. They gained $300. What is each one's part of the gain?

$180 A's stock.
240 B's stock.
480 C's stock.
$900 whole stock.

$\frac{180}{900}=\frac{1}{5}$ =A's part of the entire stock.
$\frac{240}{900}=\frac{4}{15}$=B's " " " " "
$\frac{480}{900}=\frac{8}{15}$=C's " " " " "

Hence, A must have $\frac{1}{5}$ of $300= $60 for his gain.
B " " $\frac{4}{15}$ of $300= $80 " " "
C " " $\frac{8}{15}$ of $300=$160 " " "
$300

From the above we may deduce the following

RULE.

Make each partner's stock the numerator of a fraction, and the sum of their stock a common denominator; then multiply the whole gain or loss by each of these fractions, for each partner's share.

2. Five persons, A, B, C, D, and E, are to share between them \$2400. A is to have $\frac{1}{6}$; B is to have $\frac{1}{4}$; C is to have $\frac{3}{8}$; D and E are to divide the remainder in proportion to the numbers 5 and 7. How much does each one receive?

$$\begin{array}{lllr}
\text{A receives} & \frac{1}{6} \text{ of } & \$2400 = & \$400. \\
\text{B} \quad " & \frac{1}{4} \text{ of } & 2400 = & 600. \\
\text{C} \quad " & \frac{3}{8} \text{ of } & 2400 = & \underline{900.} \\
 & & & \$1900.
\end{array}$$

$$\begin{array}{r}
\$2400 \\
\underline{1900} \\
\$500
\end{array} \text{ sum of D's and E's part.}$$

5 represents D's part.
7 represents E's part.
12 represents the sum.

Hence, D must receive $\frac{5}{12}$ of \$500 = \$208·33⅓.
E must receive $\frac{7}{12}$ of 500 = 291·66⅔.

3. There are three horses belonging to three men, employed to draw a load of plaster a certain distance for \$26·45. It is estimated that A's and B's horses do $\frac{3}{4}$ of the labor; A's and C's horses $\frac{9}{10}$; B's and C's horses $\frac{13}{20}$. They are to be paid proportionally according to these estimates. What ought each man to receive?

A's and B's horses do $\frac{3}{4} = \frac{15}{20}$.
A's and C's horses do $\frac{9}{10} = \frac{18}{20}$
B's and C's horses do $\frac{13}{20} = \frac{13}{20}$.

Adding all these fractions together, we shall obtain twice what they all do, according to the above estimate; if we take half the sum, it will give the part supposed to be done by all.

Hence, A's, B's, and C's horses do $\frac{23}{20}$.

If from this fraction we subtract $\frac{13}{20}$, which is B's and C's, we find $\frac{10}{20}$ for the part supposed to be done by A's horse. In the same way we find $\frac{5}{20}$ for the part done by B's horse. $\frac{8}{20}$ will represent the part done by C's horse.

Therefore, the parts which the three horses are supposed to do are $\frac{10}{20}$, $\frac{5}{20}$, $\frac{8}{20}$. These fractions, having a common denominator, must be to each other as their numerators, that is, as 10, 5, 8, whose sum is 23.

Hence, A ought to have $\frac{10}{23}$ of \$26·45=\$11·50.
B ought to have $\frac{5}{23}$ of 26·45= 5·75.
C ought to have $\frac{8}{23}$ of 26·45= 9·20.
Proof, \$26·45.

4. A, B, and C, agree to contribute \$365 towards building a church, which is to be at the distance of 2 miles from A, 2⅞ miles from B, and 3½ from C. They agree that their shares shall be proportional to the reciprocals of their distances from the church. What ought each to contribute?

The reciprocals of the numbers 2, 2⅞, 3½, are $\frac{1}{2}$, $\frac{8}{23}$, $\frac{2}{7}$; these reduced to a common denominator, become $\frac{161}{322}$, $\frac{112}{322}$, $\frac{92}{322}$. Now, we must obviously divide \$365 in the proportion of these numerators; their sum is 365.

Hence, A must contribute $\frac{161}{365}$ of \$365=\$161.
B " $\frac{112}{365}$ of 365= 112.
C " $\frac{92}{365}$ of 365= 92.
Proof \$365.

5. A person wills to his two sons and a daughter, the following sums: To the elder son \$1200, to the younger son \$1000, and to his daughter \$600; but it is found that

his whole estate amounts to only $800. How much ought each child to receive?

Ans. The elder son received $342·857$\frac{1}{7}$.
The younger son received 285·714$\frac{2}{7}$.
The daughter received 171·428$\frac{4}{7}$.

6. Four persons, A, B, C and D together contribute $500 towards the erection of a school-house, which is located at the distance of $\frac{1}{4}$ of a mile from A's residence, $\frac{1}{2}$ of a mile from B's, $\frac{3}{4}$ of a mile from C's, and 1 mile from D's. They contributed in the reciprocal ratio of their respective distances from the school-house. How much did each give?

Ans. A gave $\frac{12}{25}$ of $500 = $240.
B " $\frac{6}{25}$ of 500 = 120.
C " $\frac{4}{25}$ of 500 = 80.
D " $\frac{3}{25}$ of 500 = 60.

DOUBLE FELLOWSHIP.

125. WHEN the stock of the several partners continues in trade for unequal periods of time, the profit or loss must be apportioned with reference both to the stock and time. In such cases the fellowship is called DOUBLE FELLOWSHIP.

What is Double Fellowship?

EXAMPLES.

1. Three partners, A, B and C, put into trade money as *follows:* A put in $400 for 2 months; B put in $300 for

4 months; C put in $500 for 3 months. They gained $350. How must they share of this gain?

It is evident that $400 for 2 months is the same as $400 × 2 = $800 for one month.

And $300 for 4 months is the same as $300 × 4 = $1200 for one month.

And $500 for 3 months is the same as $500 × 3 = $1500 for one month.

Hence, $800 A's money for one month.
1200 B's money for one month.
1500 C's money for one month.
$3500

Therefore, by Single Fellowship,

A must have $\frac{800}{3500} = \frac{8}{35}$ of $350 = $80.
B " " $\frac{1200}{3500} = \frac{12}{35}$ of 350 = 120.
C " " $\frac{1500}{3500} = \frac{3}{7}$ of 350 = 150.
$350 Proof.

RULE.

Multiply each partner's stock by the time it was in trade; make each product the numerator of a fraction, and the sum of the products a common denominator; then multiply the whole gain or loss by each of these fractions, for each partner's share.

Repeat this Rule.

2. Three farmers hired a pasture for $55·50 for the season. A put in 6 cows for 3 months, B put in 8 cows for 2 months, C put in 10 cows for 4 months. What rent ought each to pay?

Ans. A ought to pay $13·50.
B " 12·00.
C " 30·00.

3. On the first day of January, A began business with \$650; on the first day of April following, he took B into partnership with \$500; on the first day of next July, they took in C with \$450; at the end of the year they found they had gained \$375. What share of the gain had each?

$$\textit{Ans.}\begin{cases}\text{A had \$195.}\\ \text{B \quad `` \quad 112·50.}\\ \text{C \quad `` \quad 67·50.}\end{cases}$$

4. A, B and C, have together performed a piece of work for which they receive \$94. A worked 12 days of 10 hours each; B worked 15 days of 6 hours each; C worked 9 days of 8 hours each. How ought the \$94 to be divided between them?

A worked $12 \times 10 = 120$ hours.
B " $15 \times 6 = 90$ hours.
C " $9 \times 8 = 72$ hours.
282

Therefore, A had $\frac{120}{282}$ of \$94 $= \frac{20}{47}$ of \$94 $=$ \$40.
B had $\frac{90}{282}$ of 94 $= \frac{15}{47}$ of 94 $=$ 30.
C had $\frac{72}{282}$ of 94 $= \frac{12}{47}$ of 94 $=$ 24.

5. A ship's company take a prize of \$4440, which they agree to divide among them according to their pay and the time they have been on board. Now the officers and midshipmen have been on board 6 months, and the sailors 3 months; the officers have \$12 per month, the midshipmen \$8, and the sailors \$6 per month; moreover, there are 4 officers, 12 midshipmen, and 100 sailors. What will each one's share be?

$$\textit{Ans.}\begin{cases}\text{Each officer must have \$120.}\\ \text{Each midshipman \quad `` \quad 80.}\\ \text{Each sailor \quad `` \quad 30.}\end{cases}$$

ASSESSMENT OF TAXES.

126. TAXES are moneys paid by the people for the support of government. They are assessed on the citizens in proportion to their property; except the *poll tax*, which is so much for each individual, without regard to his property.

In order to ascertain what each individual ought to pay, an accurate inventory of all the taxable property must be made.

When a tax is to be assessed on property and polls, we must first see how much the polls amount to, which must be deducted from the whole sum to be raised; we must then apportion the remainder according to each individual's property.

To effect this apportionment, we find what per cent. of the whole property to be taxed, the sum to be raised is; we then multiply each one's inventory by this per cent., expressed in decimals, and the product is his tax.

Assessors find it convenient to form a table which shall at once give the taxes on small sums, from one dollar and upwards.

What are taxes? How are they assessed? What is a poll tax? Why must an accurate inventory of all the taxable property be made? When a certain tax is to be laid on property and polls, which must be found first? Having deducted the amount of poll taxes, how do we proceed? Having found the tax on one dollar, how do we obtain the tax for any other amount? May the labor be shortened by means of a table?

EXAMPLES.

1. Suppose a tax of $600 is to be raised in a town containing 60 polls. If the whole taxable property amounts

to \$37000, and each poll tax is \$0·75, what will be A's tax, whose property is assessed at \$653, and who pays one poll?

\$0·75 amount of one poll tax.
60

\$45·00 amount of all the poll taxes.

\$600 whole amount to be raised.
Deduct 45 amount of poll taxes.

\$555 amount to be raised on \$37000.

Hence, $\frac{555}{37000}$=\$0·015 tax on one dollar.

Having found the tax on one dollar, we readily construct this

TABLE.

\$1	pays	\$0·015	\$20	pays	\$0·30	\$200	pays	\$3 00
2	"	·03	30	"	·45	300	"	4·50
3	"	·045	40	"	·60	400	"	6·00
4	"	·06	50	"	·75	500	"	7·50
5	"	·075	60	"	·90	600	"	9·00
6	"	·09	70	"	1·05	700	"	10·50
7	"	·105	80	"	1·20	800	"	12·00
8	"	·12	90	"	1·35	900	"	13·50
9	"	·135	100	"	1·50	1000	"	15·00
10	"	·15						

Now, to find A's tax, his property being \$653, I find by the above Table, that

The tax on \$600 is \$9·00.
The tax on 50 is ·75.
The tax on 3 is ·045.

The tax on \$653 is \$9·795.
One poll is ·75.

\$10·545 tax required.

2. By the above table, what would be the tax on $425, there being no poll tax? *Ans.* $6·375.

3. By the same table, what must B pay, who has 2 polls, and whose real and personal property is assessed at $762? *Ans.* $12·93.

4. If C pays 3 polls, and is assessed at $1250, how much ought he to pay? *Ans.* $21.

5. What is the tax on $375, there being no polls? *Ans.* $5·625.

6. How much is the tax on $1875? *Ans.* $28·125.

7. How much is the tax on $1100? *Ans.* $16·50.

NOTE.—By this method school rates may be computed, taxes for building school-houses, or, indeed, rates for any other similar purposes.

EQUATION OF PAYMENTS.

127. EQUATION OF PAYMENTS is a process by which we ascertain the average time for the payment of several sums due at different times.

What is Equation of Payments?

Suppose I owe $1000, of which $100 is due in 2 months, $250 in 4 months, $350 in 6 months, and $300 in 9 months. Now, if I pay the whole sum at once, how many months credit ought I to have?

A credit on $100 for 2 months is the same as a credit on $1 for *200 months.* } 100 × 2*mo.* = 200*mo.*

A credit on $250 for 4 months is the same as a credit on $1 for 1000 months.	$250 \times 4mo.$	$= 1000mo.$
A credit on $350 for 6 months is the same as a credit on $1 for 2100 months.	$350 \times 6mo.$	$= 2100mo.$
A credit on $300 for 9 months is the same as a credit on $1 for 2700 months.	$300 \times 9mo.$	$= 2700mo.$
	$1000	6000*mo.*

Hence, I ought to have the same as a credit on $1 for 6000 months. But if I wish a credit on $1000 instead of $1, it ought evidently to be for only one thousandth part of 6000 months, which is 6 months.

Hence, we infer this

RULE.

Multiply each sum by the time that must elapse before it becomes due; divide the amount of these products by the amount of the sums; the quotient will be the equated time.

EXAMPLES.

1. I purchased a bill of goods amounting to $1500, of which I am to pay $300 in 2 months, $500 in 4 months, and the balance in 6 months. What would be the mean time for the payment of the whole?

Ans. $4\frac{8}{15}$*mo.*, or 4*mo.* 16*da.*

2. A merchant owes $500 to be paid in 6 months, $600 to be paid in 8 months, and $400 to be paid in 12 months. What is the equated time of payment?

Ans. $8\frac{2}{5}$mo., or 8mo. 12*da.*

3 A owes B a certain sum; one third is due in 6 months, one fourth in 8 months, and the remainder in 12 months. What is the mean time of payment?

It is evident that it makes no difference what the amount is which A owes B, since it is certain fractional parts which become due at particular times. If we suppose the sum to be $1, then our work will be

mo. *mo.*

$\frac{1}{3} \times 6 = 2$

$\frac{1}{4} \times 8 = 2$

Remainder is $\frac{5}{12}$, and $\frac{5}{12} \times 12 = 5$

Ans. 9 months.

The least sum which we can take so as to avoid fractions is $12. In this case we have

$\frac{1}{3}$ of 12 = 4, and 4 × 6*mo.* = 24*mo.*

$\frac{1}{4}$ of 12 = 3, " 3 × 8*mo.* = 24*mo.*

Remainder, = 5, " 5 × 12*mo.* = 60*mo.*

12 108*mo.*

Hence, $\frac{108}{12} = 9$ months for the time.

4. A merchant has due him $300 to be paid in 2 months; $800 to be paid in 5 months; $400 to be paid in 10 months. What is the equated time for the payment of the whole? *Ans.* $5\frac{11}{15}$*mo.*, or 5*mo.* 22*da.*

5. A merchant owes $1200, payable as follows: $200 in 2 months, $400 in 5 months, and the remainder in 8 months. He wishes to pay the whole at one time. What is the equated time of such payment? *Ans.* 6 months.

6. A merchant bought goods to the amount of $2400, for one fourth of which he was to pay cash at the time of receiving the goods, one third in 6 months, and the balance in 10 months. What was the equitable time for the payment of the whole?

¼ of $2400=$600, which for 0
months gives 600 × 0 = 0*mo.*
⅓ of $2400=$800, which for 6
months gives 800 × 6 = 4800*mo*
Balance=$1000, which for 10
months gives 1000 × 10 = 10000*mo*
2400 14800*mo*

Hence, 14800*mo.* ÷ 2400 = 6⅙ months for the time sought.

It is obvious that the time may be estimated in days as well as in months. To illustrate this we will give several examples of this kind.

7. Suppose I owe $100 payable on January 1st, $150 on February 5th, $300 on April 10th. If we count from January 1st, and allow 29 days to February, it being Leap year, on what day ought the whole sum in equity to be paid?

Counting from January 1st, the $100 will have no time
of credit: 100 × 0*da.* = 0*da.*
From Jan. 1st to Feb. 5th is
35 days: 150 × 35*da.* = 5250*da.*
From Jan. 1st to April 10th is
100 days: 300 × 100*da.* = 30000*da.*
550 35250*da.*

Hence, 35250*da.* ÷ 550 = $64\frac{1}{11}$ days, or counted from Jan. 1st, gives March 5th for the equated time of the payment of the whole.

NOTE.—The table under ART. 76 will be found very convenient for determining the number of days between the different dates.

8. A merchant bought a bill of goods amounting to $1000. He agrees to pay $250 the first day of the next *March*, $250 on the 3d of the following May, $250 on the

4th of the following July, and the remaining $250 on the 15th of the following September. What would be the equitable time for paying the whole?

In this example, all the payments being equal, we may take for each one any sum we please. For simplicity we will consider each payment as $1.

Counting from March 1st, we see that the first payment has no credit: 1×0 days $=$ 0 days.

From March 1st to May 3d is 63 days: 1×63 days $=$ 63 days.

From March 1st to July 4th is 125 days: 1×125 days $=$ 125 days.

From March 1st to Sept. 15th is 198 days: 1×198 days $=$ 198 days.

$4 386 days.

Hence, 386 days ÷ 4 = 96½ days. Calling this 97 days, and counting from March 1st, we have June 6th for the time sought.

When a debt due at some future period has received several partial payments before the time due, to find how long beyond this time the balance may in equity remain unpaid.

9. Suppose $1000 to be due at the end of 6 months; that 3 months before it is due there was paid $100, and that 1 month before the expiration of the 6 months, there was paid $300. How long after the end of the 6 months may the balance of $600 remain unpaid?

$$\begin{array}{l} 100 \times 3\textit{mo.} = 300\textit{mo.} \\ \underline{300 \times 1\textit{mo.} = 300\textit{mo.}} \\ \qquad 600)\underline{600\textit{mo.}} \end{array}$$

Ans. 1 month.

Hence we have this

RULE.

Multiply each payment by the time it was paid before due; then divide the sum of the products thus obtained by the balance remaining unpaid; the quotient will be the equated time.

EXAMPLES.

10. Suppose $1496·41 to be due at the end of 90 days, that 84 days before it is due there is paid $500; 32 days before the 90 days expire there is paid $502·50. How long after the 90 days before the balance of $493·91 ought to be paid? *Ans.* 117½ days.

11. A lent $200 to B for 8 months; at another time he lent him $300 for 6 months. For how long a time ought B to lend A $800 to balance the favor?

Ans. 4¼ months.

128. It is customary with many merchants to give a credit, of from 3 to 6 months, on their bills of sale. In such cases, in settling up their accounts, which generally consist of various items of debit and credit at sundry times, it is very desirable to have some simple rule by which the cash balance can be found. We have prepared a rule for this purpose.

Suppose A has the following account with B:

1848.		*Dr.*	1848.		*Cr.*
Jan. 10.	To Merchandise, . .	$100	Feb. 8.	By Merchandise, .	$50
March 26.	" "	400	April 23.	" "	375

What is the *cash* balance, July 10, 1848, if interest is estimated at 7 per cent., and a credit of 30 days is allowed *on all* the different sums?

If interest were not considered, the above account could be balanced as follows:

1848.		*Dr.*	1848.		*Cr.*
Jan. 10.	To Merchandise, . .	$100	Feb. 8.	By Merchandise, . .	$50
March 26.	" "	400	April 23.	" "	375
				" Balance	75
		$500			$500
	To Balance	$75			

Had no credit been given, the debits should be increased by the following items of interest: (See Table, Art. **76**, and Rule, Art. **114**.)

On $100 for 182 days at 7 per cent. $= 100 \times 182 \times \frac{.07}{365}$.
" 400 " 106 " " " $= 400 \times 106 \times \frac{.07}{365}$.

In like manner the credits should be increased by interest:

On $50 for 153 days at 7 per cent. $= 50 \times 153 \times \frac{.07}{365}$.
" 375 " 78 " " " $= 375 \times 78 \times \frac{.07}{365}$.

But, since 30 days credit is given on all sums, it follows that by the above, we should increase the debits by an excess of interest equal to the interest of the sum of debits, $500, for 30 days $= 500 \times 30 \times \frac{.07}{365}$. In like manner we should increase the credits by an excess of interest equal to the interest of sum of credits, $425, for 30 days $= 425 \times 30 \times \frac{.07}{365}$.

Now if, instead of diminishing the debit items of interest by $500 \times 30 \times \frac{.07}{365}$, and the credit items of interest by $425 \times 30 \times \frac{.07}{365}$, we merely diminish the debit items of interest by the interest on *merchandise* balance, $75, for 30 days, which is $75 \times 30 \times \frac{.07}{365}$, the result will be the same. And since taking any sum from one side of a book account has the same effect as adding the same sum to *the other* side, it follows, that instead of diminishing the debit items of interest by $75 \times 30 \times \frac{0{\cdot}07}{365}$, we may increase the credit items of interest by this same quantity.

From which we see that the difference between $100 \times 182 \times \frac{0{\cdot}07}{365} + 400 \times 106 \times \frac{0{\cdot}07}{365}$ and $50 \times 153 \times \frac{0{\cdot}07}{365} + 375 \times 78 \times \frac{0{\cdot}07}{365} + 75 \times 30 \times \frac{0{\cdot}07}{365}$ is the *interest* balance.

The operations indicated in the foregoing work may be exhibited in a more condensed form, as follows :

Debits.			Credits.		
$	Days.		$	Days.	
100 ×	182 =	18200	50 ×	153 =	7650
400 ×	106 =	42400	375 ×	78 =	29250
		60600	75 ×	30 =	2250
		39150			39150

$\frac{0{\cdot}07}{365}$ of $\overline{21450}$ = $4·11 = *interest* balance.

Hence the foregoing account will become balanced as follows :

1848.		*Dr.*	1848.		*Cr.*
Jan. 10.	To Merchandise, .	$100·00	Feb. 8.	By Merchandise, . .	$50·00
March 26.	" " . . .	400·00	April 23.	" "	375·00
July 10.	" balance of interest	4·11	July 10.	" balance	79·11
		504·11			$504·11
July 10.	" Cash balance . .	$79·11			

From the above, we deduce this

RULE.

Place such sum on the debtor or credit side as may be necessary to balance the account, which sum may be regarded as MERCHANDISE BALANCE. *Then multiply the number of dollars in each entry by the number of days from the time such entry was made, to the time of settlement; observing to multiply the merchandise balance by the number of days for which credit is given.*

Multiply the difference between the sum of the debit products, and the sum of the credit products, by the interest of $1 for 1 day; the product will be the number of dollars in INTEREST BALANCE, *which will be in favor of the debit side of account, when the sum of debit products exceeds the sum of credit products; but in favor of the credit side when the sum of credit products exceeds the sum of debit products. If, then, the interest balance be added to, or subtracted from, the merchandise balance, as the case may require, it will give the cash balance.*

EXAMPLES.

1. Suppose A has the following account with B:

1848.		*Dr.*	1848.		*Cr.*
Jan. 1.	To Merchandise, . . .	$200	Jan. 15.	By Merchandise, . .	$300
March 3.	" "	500	March 20.	" "	400
May 10.	" "	100	May 3.	" "	200
June 6.	" "	300	July 1.	" "	50
		1100			$950
		950			
	Merchandise balance	$150			

What is the cash balance of the above account on the 1st of July, 1848, provided each individual is allowed 90 days time on his purchases, if interest is estimated at 7 per cent.?

OPERATION.

Debit Products.	Credit Products.
$ Days.	$ Days.
200 × 182 = 36400	300 × 168 + 50400
500 × 120 = 60000	400 × 103 = 41200
100 × 52 = 5200	200 × 59 = 11800
300 × 25 = 7500	50 × 0 = 0
1100 × 109100	Md. bal. 150 × 90 = 13500
	116900
	109100

24 $\frac{.07}{360}$ of 7800=1·49 nearly.

hence $1·49 is the *interest* balance, which balance is in favor of the credit side; but $150, the *merchandise* balance, was in favor of the debtor side; consequently the *cash* balance is $150—$1·49=$148·51 in favor of A.

2. Suppose A's account with B to have been as follows:

1848.		*Dr.*	1848.		*Cr.*
Jan. 10.	To Merchandise, . .	$250·37	June 25.	By Merchandise, . .	$37·51
Feb. 25.	" "	113·04	July 20.	" "	50·98
March 1.	" "	405·59	July 28.	" "	600·03
		769·00			$688·52
		688·52			
	Merchandise balance	80·48			

What is the cash balance, and in whose favor, on the 1st of August, 1848, provided 6 months, or 180 days' time is given, interest being 6 per cent.?

NOTE.—In practice, when the cents in any of the entries, as in this example, are less than 50, we may, without sensible error, omit them; but when they are 50, or greater, we may consider them as an additional dollar.

OPERATION.

DEBIT PRODUCTS.		CREDIT PRODUCTS.	
$ Days.		$ Days.	
250 × 204 =	51000	38 × 37 =	1406
113 × 158 =	17854	51 × 12 =	612
406 × 153 =	62118	600 × 4 =	2400
769	130972	Md. bal. 80 × 180 =	14400
	18818		18818

$\frac{\cdot 06}{365}$ of 112154=18·44 nearly; hence $18·44 is the *interest* balance, which balance is in favor of the *debtor* side. The *merchandise* balance of $80·48 was also in favor of the debtor side, consequently the *cash* balance is $80·48+$18·44=$98·92 in favor of A.

What is meant by a cash balance? What is meant by merchandise balance? Instead of diminishing one side of a book account by a certain sum, what may be done? How is the interest balance found? In favor of which side of an account will the interest balance be? Repeat the Rule. In practice, what may be done with the cents in any of the entries?

INVOLUTION.

129. The product arising from multiplying a number into itself is called the *second power*, or the *square* of that number. Thus, $3\times3=9$: the number 9 is the square of 3.

If the square of a number be again multiplied by that number, the result is called the *third power*, or the *cube* of the number. Thus, $3\times3\times3=27$: the number 27 is the cube of 3.

The word *power* denotes the product arising from multiplying a number into itself a certain number of times; and the number thus multiplied is called the *root*. Thus, 9 is the second power of 3, and 3 is the square root of 9. In the same manner 27 is the third power of 3, and 3 is the cube root of 27.

The product arising from multiplying a number into itself is called what? If it be used as a factor three times, what power is it? The number 9 is what power of 3? The number 27 is what power of 3? What is the square root of 9? What is the cube root of 27?

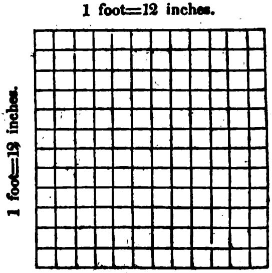
130. *Involution is the method of finding the powers of numbers.*

To denote that a number is to be raised to a power, a small figure is placed above, a little to the right of the number whose power is to be found.

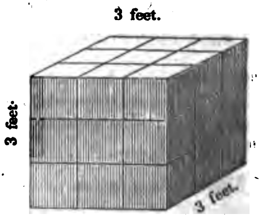
The small figure is called the *index*, or *exponent.*

Thus, $4^2 = 4 \times 4 = 16$; here the exponent is 2, and 4^2 denotes the second power of 4. In the same way we have

$3^1 = 3$ the first power of 3.
$3^2 = 3 \times 3 = 9$ the second power of 3.
$3^3 = 3 \times 3 \times 3 = 27$ the third power of 3.
$3^4 = 3 \times 3 \times 3 \times 3 = 81$ the fourth power of 3.
$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$ the fifth power of 3.
&c., &c.



The *second* power of a number is called the *square* of that number, because it may be represented by means of a geometrical square. Thus, in the adjacent figure if the side of this square is 12 *linear* units, as 12 inches long, its entire surface will be denoted by $12 \times 12 = 144$ *square* units, which in this case will be 144 square inches.



For a similar reason, the third power of a number is called the *cube* of that number, since it can be represented by the geometrical cube, as in the adjacent figure, where the side of the cube is supposed to be 3 *linear* feet, consequently each *face* will be $3 \times 3 = 9$ *square feet,* and its volume will be $3 \times 3 \times 3 = 27$ cubic feet.

To raise a number to any power, we have the following

RULE.

Multiply the number continually by itself, as many times less one as there are units in the exponent; the last product will be the power sought.

What is Involution? How do we denote that a number is to be raised to a power? What is this small figure placed above, a little to the right, called? Repeat the Rule.

EXAMPLES.

1. What is the square of 23? *Ans.* 529.
2. What is the cube of 17? *Ans.* 4913.
3. What is the fifth power of 47? *Ans.* 229345007.
4. What is the ninth power of 9? *Ans.* 387420489.
5. What is the square of 625? *Ans.* 390625.
6. What is the cube of 48? *Ans.* 110592.
7. What is the square of 0·75? *Ans.* 0·5625.
8. What is the cube of 0·65? *Ans.* 0·274625.
9. What is the square of $8\frac{1}{2}$? *Ans.* $72\frac{1}{4}$.
10. What is the square of $\frac{3}{4}$? *Ans.* $\frac{9}{16}$.
11. What is the cube of $\frac{7}{8}$? *Ans.* $\frac{343}{512}$.
12. What is the cube power of $3\frac{1}{3}$? *Ans.* $\frac{1000}{27}=37\frac{1}{27}$.
13. What is the fifth power of $2\frac{3}{4}$?
Ans. $\frac{161051}{1024}=157\frac{283}{1024}$.
14. What is the third power of 0·5? *Ans.* 0·125.
15. What is the fourth power of 0·25?
Ans. 0·00390625.
16. What is the square of $\frac{4}{5}$? *Ans.* $\frac{16}{25}$
17. What is the cube of $1\frac{1}{2}$? *Ans.* $3\frac{3}{8}$.
18. *What is the cube of* $2\frac{1}{5}$? Ans. $10\frac{81}{125}$.

EVOLUTION.

131. EVOLUTION is the reverse of involution; that is, it explains the method of resolving a number into equal factors.

When a number can be resolved into equal factors, one of these factors is called a *root* of the number.

If the number is resolved into *two* equal factors, one of these factors is called the *square root.*

Thus, $36=6\times6$, and 6 is the square root of 36. In the same way 7 is the square root of 49, since $49=7\times7$.

To denote that the square root of a number is to be found, we use the symbol $\sqrt{}$. Thus, $\sqrt{81}$ denotes that the square root of 81 is to be found; that is, $\sqrt{81}=9$; so $\sqrt{100}=10$; $\sqrt{25}=5$.

When a number is resolved into *three* equal factors, one of these factors is called the *cube root* of the number.

Thus, $64=4\times4\times4$, and 4 is the cube root of 64; also 5 is the cube root of 125, since $125=5\times5\times5$.

To indicate that the cube root of a number is to be found, we use the symbol $\sqrt[3]{}$; thus, $\sqrt[3]{27}$ denotes that the cube root of 27 is to be found; that is, $\sqrt[3]{27}=3$; so $\sqrt[3]{64}=4$; $\sqrt[3]{8}=2$; $\sqrt[3]{216}=6$.

We shall hereafter use the dot (.) to denote multiplication. Thus 3.4 indicates that 3 is to be multiplied by 4. Also 3×4.8 denotes that the product of 3 and 4 is to be multiplied by 8.

When the dot is used to denote multiplication, it is placed near the bottom of the line, but when used to denote a decimal, it is placed near the middle of the line.

What is Evolution? When a number can be resolved into a number of equal factors, what is such a factor called? If the number is resolved into two equal factors,

what is the root called? When resolved into three equal factors, what is the root called? What character is used to denote the square root? What to denote the cube root? What is the square root of 81? What is the square root of 100? What is the cube root of 27? What is the cube root of 8? What additional sign of multiplication is used?

Before explaining the method of extracting the square roots of numbers, we shall involve some numbers by considering them as decomposed into *units*, *tens*, *hundreds*, &c.

What is the square of 25? Of 35?

OPERATION.

$$\begin{array}{r} 25=20+\ \ 5 \\ 20+\ \ 5 \\ \hline 100+25 \\ 400+100 \\ \hline 25^2=400+200+25 \\ \hline \end{array}$$

OPERATION.

$$\begin{array}{r} 35=30+\ \ 5 \\ 30+\ \ 5 \\ \hline 150+25 \\ 900+150 \\ \hline 35^2=900+300+25 \\ \hline \end{array}$$

By a similar method, we find

$46^2=(40+6)^2=40^2+2\times40.6+6^2=1600+480+36.$

$54^2=(50+4)^2=50^2+2\times50.4+4^2=2500+400+16.$

$93^2=(90+3)^2=90^2+2\times90.3+3^2=8100+540+9.$

$48^2=(40+8)^2=40^2+2\times40.8+8^2=1600+640+64.$

From the above, we draw the following property:

The square of the sum of two numbers is equal to the square of the first number, plus twice the product of the first number into the second, plus the square of the second.

If we wish the square of the sum of three numbers, as 6+8+9; we may unite the first and second by means of a parenthesis, thus, for 6+8+9, we may make use of (6+8)+9; and now regarding 6+8 as one number, the preceding rule for the sum of two numbers will apply to (6+8)+9, that is, the square of 6+8+9 is equal to the square of (6+8,) plus twice the product of (6+8) into 9, plus the square of 9. But the square of 6+8 has already been shown to be, the square of 6, plus twice the product of 6 into 8, plus the square of 8. Hence, the square of 6+8+9 is equal to the square of 6, plus twice the product of 6 into 8, plus the square of 8, plus twice the product of the sum of 6 and 8 into 9, plus the square of 9. Or in general terms,

The square of the sum of three numbers is equal to the square of the first number, plus twice the product of the first number into the second, plus the square of the second; plus twice the product of the sum of the first two into the third, plus the square of the third.

Continuing in this way, we could show that, *the square of the sum of any number of numbers is the square of the first number, plus twice the product of the first number into the second, plus the square of the second; plus twice the product of the sum of the first two into the third, plus the square of the third; plus twice the product of the sum of the first three into the fourth; plus the square of the fourth; plus twice the product of the sum of the first four into the fifth, plus the square of the fifth; and so on.*

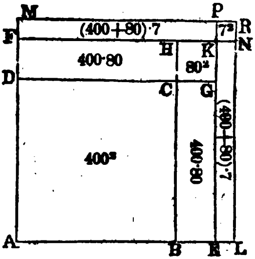
We will now apply this general rule to a few examples.

1. $(2+3)^2=2^2+2\times 2.3+3^2$.
2. $(5+7)^2=5^2+2\times 5.7+7^2$.
3. $(3+4+5)^2=3^2+2\times 3.4+4^2+2\times(3+4).5+5^2$.
4. $(5+6+7)^2=5^2+2\times 5.6+6^2+2\times(5+6).7+7^2$.
5. $(7+8+9)^2=7^2+2\times 7.8+8^2+2\times(7+8).9+9^2$.
6. $(35)^2=(30+5)^2=30^2+2\times 30.5+5^2$.
7. $(47)^2=(40+7)^2=40^2+2\times 40.7+7^2$.
8. $(365)^2=(300+60+5)^2=300^2+2\times 300.60+60^2+2\times(300+60).5+5^2$.

9. $(487)^2=(400+80+7)^2=400^2+2\times400.80+80^2+2\times(400+80).7+7^2$.

The above method of squaring a number consisting of the sum of two or more numbers, is elegantly illustrated geometrically as follows:

The square ABCD may be enlarged to the square AEKF, by the addition of the two equal rectangles BG and DH, whose lengths are each equal to the side AB of the original square, and whose widths are equal to BE, the quantity by which the side of the square has been augmented, also a little square, CGKH, whose side is the same as the width of one of the equal rectangles.



Again, the square AEKF, having its side increased by EL, becomes augmented by the two rectangles EN, FP, and the little square KR. Thus we might continue to augment the square last found by the addition of two equal rectangles, and a little square; the length of each rectangle being equal to the side of the square which is to be augmented, and the width equal to the quantity by which the side of the square is increased; and the side of the little square being the same as the width of one of the rectangles. The diagram is adapted to the case of squaring 400+80+7=487.

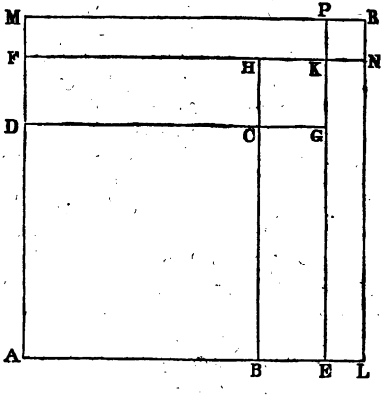
132. Let us now, by reversing the above process, deduce a rule for extracting the square root.

Let it be required to extract the square root of 527076. For the sake of simplicity, we will suppose we are required to find the number of feet in the side of a square whose area shall contain 527076 square feet.

The smallest number, consisting of two figures, which is 10, becomes, when squared, 100; having more than two figures. Again, the largest number of two figures, 99, becomes, when squared, 9801, having four figures. Hence, when a number consists of more than two figures, and of not more than four, its square root will consist of two figures. By a similar method it may be shown, that when a number consists of more than four, and of not more than six figures, its square root will consist of three figures. Therefore, if we separate a number into groups of two figures each, the number of groups will denote the number of figures in the square root of that number.

In this example, we see that there must be three figures in the root.

We know that the side of the square sought must exceed 700 *linear* feet, since the square of 700 is 490000, which is less than 527076; we also know that the side of this square must be less than 800 *linear* feet, since the square of 800 is 640000, which is greater than 527076. Hence the first, or hundreds' figure of the root, is 7; which is the greatest number whose square can be contained in 52, the first or left-hand period.



If we suppose each side of the square ABCD to be 700 *linear* feet, its surface will be 700 × 700= 490000 *square* feet, which, subtracted from 527076 *square* feet, leaves 37076 *square* feet.

Hence it is necessary to increase the square ABCD, by 37076 *square* feet. We have seen that such increase is effected by the addition of two equal *rectangles*, and a little square. The surface of the two *rectangles* will evidently make by far the largest portion of the whole increase. The length of one of these rectangles is the same as the length of a side of the square ABCD, which has already been shown to be 700 *linear* feet. The length of the two rectangles taken together will be twice 700 *linear* feet, or what would be the same thing, 700 *linear* feet added to 700 *linear* feet. If to BC, which is 700 *linear* feet, we add CD, which is also 700 *linear* feet, we shall have BC+CD equal to 1400 *linear* feet, for the length of the two rectangles. Were we to multiply 1400 by the width of a rectangle, we should obtain the number of *square* feet in these rectangles, or nearly the 37076 *square* feet which require to be added. Consequently, if we divide 37076 by 1400, the quotient will give the approximate width of the *rectangles*. Using 1400 as a *trial divisor*, we find it to be contained between 20 and 30 times in 37076; hence the second or ten's figure of the root is 2. But besides the rectangles, there must be added the *little square* CGKH, each side of which is 20 *linear* feet, we will therefore add 20 to 1400, and thus obtain 1420 for the total length of the two *rectangles* and the side of the *little* square. Now, multiplying 1420 by 20, we obtain 28400 *square* feet for the total additions, which subtracted from 37076, leaves 8676 *square* feet. The square AE KF thus completed is 720 feet on a side.

Again, a side of this square is to be further increased so that the added surface will amount to 8676 *square* feet. And, as before, the parts added will consist of two equal *rectangles* and a *little square*. The *trial* divisor, which is the sum of the length of the two new rectangles, is the same as the sum of two sides of the square AEKF.

If, now, to 1420 already found, we add 20, we shall have 1440, which is the sum of EK and KF, and which is our second *trial divisor*. We find this divisor contained between 6 and 7 times in 8676; hence our third or units' figure of the root is 6. Therefore 6 is the width of the second set of rectangles. The second *little square* KNRP, of the same width as the rectangles, must be 6 *linear* feet on a side; therefore, adding 6 to 1440, we find 1446 for the whole length of the new rectangles and a side of the second *little square*. Multiplying 1446 by 6, we obtain 8676 *square* feet as the sum of the series of additions to the square AEKF, thus forming the square ALRM, which is the square sought; each side being 726 feet.

The above work may be arranged as follows:

	NUMBER.	ROOT.
Linear feet.	*Square feet.*	*Linear feet.*
700	527076	(700+20+6=726.
1400=1st trial divisor	490000	
1420	37076	
1440=2d trial divisor	28400	
1446	8676	
	8676	
	0	

If we omit the ciphers on the right, the work will take the following condensed form:

	NUMBER.	ROOT.
Linear feet.	*Square feet.*	*Linear feet*
7	527076(	726.
14=1st trial divisor	49	
142	370	
144=2d trial divisor	284	
1446	8676	
	8676	
	0	

CASE I.

From the above process, we deduce the following rule for the extraction of the square root of a whole number.

RULE.

I. Separate the given number into periods of two figures each, counting from the right towards the left. When the number of figures is odd, it is evident that the left-hand, or first period, will consist of but one figure.

II. Find the greatest square in the first period, and place its root at the right of the number, in the form of a quotient in division, also place it at the left of the number. Subtract the square of this root from the first period, and to the remainder annex the second period; the result will be the FIRST DIVIDEND.

III. To the figure of the root, as placed at the left of the number, add the figure itself, and the sum, with a cipher annexed, will be the FIRST TRIAL DIVISOR. *See how many times this trial divisor is contained in the dividend; the quotient will be the next figure of the root; this must be added to the* TRIAL DIVISOR; *the result will be the* TRUE DIVISOR. *Multiply the true divisor by this last figure of the root, and* *subtract the product from the dividend, and to the remainder annex the next period, for a* NEW DIVIDEND.

IV. To the last TRUE DIVISOR *add the last figure of the root; the sum, with a cipher annexed, will be a new* TRIAL DIVISOR. *Continue to operate as before, until all the periods have been brought down.*

NOTE.—In case any dividend is not so great as its trial divisor, we must write 0 as the next figure of the root; we must also place 0 at the right of the divisor, and form a new dividend by annexing a new period.

EXAMPLES.

1. What is the square root of 11390625?

OPERATION.

```
3          11'39'06'25)3375
63          9
667         ---
6745        239
            189
            ----
             5006
             4669
             -----
              33725
              33725
              -----
                  0
```

2. What is the square root of 11019960576? *Ans.* 104976.

3. What is the square root of 276793836544? *Ans.* 526112.

4. What is the square root of 12321? *Ans.* 111.

5. What is the square root of 53824? *Ans.* 232.

6. What is the square root of 30858025? *Ans.* 5555.

7. What is the square root of 16983563041? *Ans.* 130321.

8. What is the square root of 852891037441 ?

Ans. 923521.

9. What is the square root of 61917364224 ?

Ans. 248832.

CASE II.

To extract the square root of a decimal fraction, or of a number consisting partly of a whole number, and partly of a decimal, we have this

RULE.

I. Annex one cipher, if necessary, to the decimals, so that their number shall be even.

II. Then point off the decimals into periods of two figures each, counting from the units' place towards the right. If there are whole numbers, they must be pointed off as in Case I. Then extract the root, as in Case I.

NOTE.—If the given number has not an exact root, there will be a remainder after all the periods have been brought down, in which case the operation may be extended by forming new periods of ciphers.

EXAMPLES.

1. What is the square root of 3486·78401 ?

Ans. 59·049.

2. What is the square root of 6·5536 ? Ans. 2·56.

3. What is the square root of 0.00390625 ?

Ans. 0·0625.

4. What is the square root of 17 ? Ans. 4·123, nearly.

5. What is the square root of 37·5 ?

Ans. 6·123, nearly.

6. What is the square root of 0·0000012321 ?

Ans. 0·00111.

CASE III.

To extract the square root of a vulgar fraction, or mixed number, we have this

RULE

I. Reduce the vulgar fraction, or mixed number, to its simplest fractional form.

II. Then extract the square root of the numerator and denominator separately, if they have exact roots; but when they have not, reduce the fraction to a decimal, and proceed as in Case II.

EXAMPLES.

1. What is the square root of $\frac{256}{324}$? *Ans.* $\frac{8}{9}$.
2. What is the square root of $\frac{10125}{12005}$? *Ans.* $\frac{45}{49}$.
3. What is the square root of $4\frac{21}{25}$? *Ans.* $2\frac{1}{5}$.
4. What is the square root of $\frac{5}{9}$ of $\frac{4}{5}$ of $\frac{4}{7}$ of $\frac{7}{9}$? *Ans.* $\frac{4}{9}$.
5. What is the square root of $4\frac{1}{9}$? *Ans.* 2·027 nearly.
6. What is the square root of $\frac{11}{17}$? *Ans.* 0·8044 nearly.
7. What is the square root of $\frac{13}{48}$? *Ans.* 0.52 nearly.

EXAMPLES INVOLVING THE PRINCIPLES OF THE SQUARE ROOT.

133. *A triangle* is a figure having three sides, and consequently three angles.

When one of the angles is right, like the corner of a square, the triangle is called a *right-angled triangle.* In this case the side opposite the right angle is called the *hypotenuse.*

It is an established proposition of geometry, that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

From the above proposition, it follows that the square of the hypotenuse, diminished by the square of one of the sides, equals the square of the other side.

By means of these properties, it follows that two sides of a right-angled triangle being given, the third side can be found.

EXAMPLES.

1. How long must a ladder be, to reach to the top of a house 40 feet high, when the foot of it is 30 feet from the house?

In this example, it is obvious that the ladder forms the hypotenuse of a right-angled triangle, whose sides are 30 and 40 feet respectively. Therefore, the square of the length of the ladder must equal the sum of the squares of 30 and 40.

$$\begin{array}{l} 30^2 = 900 \\ 40^2 = 1600 \\ \sqrt{2500} = 50, \text{ the length of the ladder.} \end{array}$$

2. Suppose a ladder 100 feet long, to be placed 60 feet from the roots of a tree, how far up the tree will the top of the ladder reach? *Ans.* 80 feet.

3. Two persons start from the same place, and go, the one due north, 50 miles, the other due west, 80 miles. How far apart are they? *Ans.* 94·34 miles, nearly.

4. What is the distance through the opposite corners of a square yard? *Ans.* 4·24264 feet, nearly.

5. The distance between the lower ends of two equal rafters, in the different sides of a roof, is 32 feet, and the

height of the ridge above the foot of the rafters is 12 feet. What is the length of a rafter? *Ans.* 20 feet.

6. What is the distance measured through the centre of a cube, from one corner to its opposite corner, the cube being 3 feet, or one yard, on a side?

Ans. 5·196 feet, nearly.

We know, from the principles of geometry, that all similar surfaces, or areas, are to each other as the squares of their like dimensions.

7. Suppose we have two circular pieces of land, the one 100 feet in diameter, the other 20 feet in diameter. How much more land is there in the larger than in the smaller?

By the above principle of geometry it follows, that the quantity of land in the two circles must be as the squares of the diameters, that is, 100^2 to 20^2, or as 25 to 1. Hence, there is 25 times as much in the one piece as there is in the other.

8. Suppose, by observation, it is found that 4 gallons of water flow through a circular orifice of 1 inch in diameter in 1 minute. How many gallons would, under similar circumstances, be discharged through an orifice of 3 inches in diameter, in the same length of time?

Ans. 36 gallons.

9. What length of thread is required to wind spirally around a cylinder, 2 feet in circumference and 3 feet in length, so as to go but once around?

It is evident that if the cylinder be developed, or placed upon a plane, and caused to roll once over, that the convex surface of the cylinder will give a rectangle, whose width is 2 feet, and length 3 feet; at the same time the thread will form its diagonal. Hence, the length of the thread is $\sqrt{4+9}=\sqrt{13}=3·60555$ feet, nearly.

EXTRACTION OF THE CUBE ROOT.

134. We will first involve a number to the third power, that is, we will find the cube of that number.

Let the number be 45.

$45^3=45\times45\times45=91125$. But we will separate this number into parts; that is, into *tens* and *units*, and show by the aid of the exponent and the symbols, how the cube of the number when thus separated is obtained.

OPERATION.

$$\begin{array}{l} 40+5 \\ \underline{40+5\qquad\qquad} \\ 40^2+40.5 \\ \underline{\quad+40.5+5^2} \\ 40^2+2\times40.5+5^2=\text{the square of } 40+5. \\ \underline{40+5\qquad\qquad\qquad\quad} \\ 40^3+2\times40^2.5+\qquad 40.5^2 \\ \underline{\quad+\qquad 40^2.5+2\times40.5^2+5^3} \\ 40^3+3\times40^2.5+3\times40.5^2+5^3=\text{cube of } 40+5. \end{array}$$

By a similar process we shall obtain

$$(6+8)^3=6^3+3\times6^2.8+3\times6.8^2+8^3.$$

That is, *the cube of the sum of two numbers is, the cube of the first number, plus three times the product of the square of the first number into the second, plus three times the product of the first into the square of the second, plus the cube of the second.*

If we wish the cube of the sum of three numbers, as $6+8+9$, we may unite the first and second by means of a parenthesis: thus, for $6+8+9$, we may make use of

(6+8)+9, and regarding (6+8) as one number, we find, according to the foregoing statement, that the cube of (6+8)+9 is equal to the cube of (6+8) plus three times the product of the square of (6+8) into 9, plus three times the product of (6+8) into the square of 9, plus the cube of 9. But the cube of 6+8, has already been shown to be equal to the cube of 6, plus three times the product of the square of 6 into 8, plus three times the product of 6 into the square of 8, plus the cube of 8. Hence the cube of 6+8+9 is equal to the cube of 6, plus three times the square of 6 into 8, plus three times 6 into the square of 8, plus the cube of 8; plus three times the square of the sum of 6 and 8 into 9, plus three times the sum of 6 and 8 into the square of 9, plus the cube of 9. *And in general, we have the cube of the sum of any number of numbers equal to the cube of the first number, plus three times the square of the first number into the second, plus three times the first into the square of the second, plus the cube of the second; plus three times the square of the sum of the first two into the third, plus three times the sum of the first two into the square of the third, plus the cube of the third; plus three times the square of the sum of the first three into the fourth, plus three times the sum of the first three into the square of the fourth, plus the cube of the fourth, and so on.*

Thus:

$$(2+3)^3=2^3+3\times2^2.3+3\times2.3^2+3^3.$$

$$(5+7)^3=5^3+3\times5^2.7+3\times5.7^2+7^3.$$

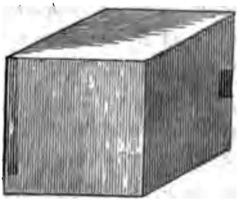
$$(5+6+7)^3=5^3+3\times5^2.6+3\times5.6^2+6^3$$
$$+3\times(5+6)^2.7+3\times(5+6).7^2+7^3.$$

$$(365)^3=(300+60+5)^3=300^3+3\times300^2.60$$
$$+3\times300.60^2+60^3+3\times(300+60)^2.5$$
$$+3\times(300+60).5^2+5^3.$$

The cubing of a number may be illustrated geometrically as follows:

Let it be required to cube 45, the number before employed. To simplify the illustration, suppose we are required to find the number of cubic inches in a cube whose side is 45 inches. Separating 45 into 40+5, we will suppose the cube, (fig. 1,) to be 40 inches on a side; then $40\times40\times40$ will give the solid contents of this cube, represented by 40^3.

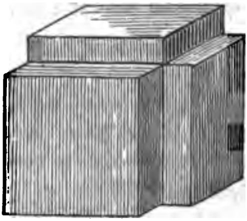
Fig. 1.



$40^3=40\times40\times40$
$=64000$

Let fig. 2 represent the cube increased by three equal *slabs*; then 3 (the number of slabs) times 40^2 (the surface of one of the slabs,) multiplied by 5, the thickness of a slab, will give the solid contents of the slabs, represented by $3\times40^2.5$.

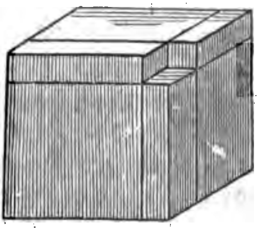
Fig. 2.



$40^2=40\times40$
$=1600$
$\times$ by 3
4800
$\times$ by 5
24000

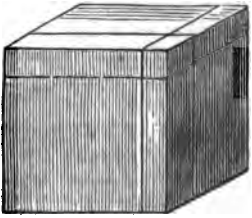
Let fig. 3 represent the solid, (as in fig. 2,) further increased by *three equal corner* pieces; then 3 (the number of corner pieces) times 40 (the length of one corner piece) multiplied into 5^2, the surface of an end of a corner piece, will give the solid contents of the corner pieces, represented *by* 3×40.5^2.

Fig. 3.



$3\times40=120$
$\times$ by $5^2=$ 25
600
240
3000

Fig. 4.



$5^3 = 5 \times 5 \times 5$
$= 125$

Let fig. 4 represent the solid (as in fig. 3) further increased by *a little corner cube*, each side of which is 5 inches; then $5 \times 5 \times 5$ will give the solid contents of this cube, represented by 5^3.

Then the *whole cube* thus increased will be represented by $45^3 = 40^3 + 3 \times 40^2.5 + 3 \times 40.5^2 + 5^3$
$= 64000 + 24000 + 3000 + 125 = 91125$.

135. We will now endeavor to deduce a rule for the extraction of the Cube Root.

Let it be required to find the cube root of 382657176.

For the sake of simplicity, we will suppose 382657176 to denote the number of cubic feet in a geometrical cube; we are required to find the number of *linear* feet in a side of this cube, that is, the length of one of its sides.

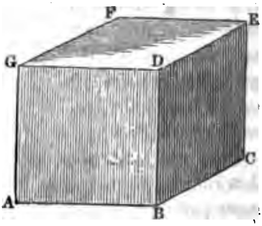
We will first inquire how many figures the root will have.

The smallest number, consisting of two figures, which is 10, becomes, when cubed, 1000, having more than three figures. Again, the largest number, 99, which consists of two figures, becomes, when cubed, 970299, which consists of six figures. Hence, when a number consists of more than three figures, and not of more than six, its cube root will consist of two figures. By a similar method it may be shown, that when a number consists of more than six, and of not more than nine figures, its cube root will consist of three figures. Therefore, if we separate a number into groups of three figures each, the number of groups will denote the number of figures in the cube root of that number.

In the present example, we know that there must be three figures in the root.

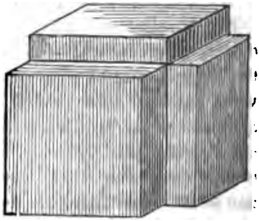
We know that the side of the cube sought must exceed 700 *linear* feet, since the cube of 700 is 343000000, which is less than 382657176, we also know that the side of this cube must be less than 800 *linear* feet, since the cube of 800 is 512000000, which is greater than 382657176. Hence the first figure of our root, or the figure in the hundred's place is 7; whose cube, 343, is the greatest cube contained in 382, the first, or left-hand period. If we suppose each side of the cube, represented by figure 1, to be 700 *linear* feet, one of the equal faces, as the upper face DEFG, will be denoted by $700\times 700=490000$ *square* feet. The solid contents of the cube will be represented by $700\times 700=490000\times 700=343000000$ *cubic* feet. Subtracting 343000000 *cubic* feet from 382657176 *cubic* feet, we find 39657176 *cubic* feet for a remainder.

Fig. 1.



Hence it is necessary to increase the cube, figure 1, by 39657176 *cubic* feet. We have seen (ART. **134**) that such increase is effected by the addition of three equal *slabs*, three equal *corner pieces*, and an additional *cube;* and that the contents of the three *slabs* will make by far the largest portion of the whole increase.

Fig. 2.



The number of square feet in the face of one of these slabs will be the same as the number of square feet in the face of the cube, figure 1, which has already been shown to be 490000 *square* feet. The surface of the three slabs will be three times 490000 *square* feet; or, which would be the same thing, twice 490000 *square* feet, added to 490000 *square* feet.[*] If to AB, (fig. 1,) which is 700 *linear* feet, we add BC, which is also 700 *linear* feet, we shall have AB+BC equal to 1400 *linear* feet, which, multiplied by DB, equal to 700 *linear* feet, will give 980000 *square* feet, for the area ABDG+BCED, which, added to DEFG, which is 490000 *square* feet, will give 1470000 square feet, for the area of three faces of the cube, figure 1, which is the same as the area of the three slabs. Were we to multiply 1470000 by the thickness of the slabs, we should obtain the *cubic* feet in these slabs. And since the contents of the slabs make nearly the whole amount added, it follows that 1470000 multiplied by the thickness of slabs, will give nearly 39657176 *cubic* feet. Consequently, if we divide 39657176 by 1470000, the quotient will give the approximate thickness of the *slabs*. Using 1470000 as a *trial divisor*, we find it to be contained between 20 and 30 times in 39657176; hence the second or tens' figure of the root is 2.

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[*] It will be noticed that the *peculiar steps* throughout this demonstration, have reference to the mode of extracting the Cube Root which follows. The object of these processes is, to make use of what has been obtained in one stage of the work for the stage next succeeding; to obtain a new quantity by *adding to* one already *in hand*, instead of *multiplying* an original quantity; thereby saving much time and labor.

We have already remarked that 1470000 multiplied by 20, the thickness of the *slabs*, will give their solid contents. But besides the *slabs* there must be added three *corner pieces*, each of which is 700 feet long, and of the same thickness as the *slabs*, that is, 20 feet. Since each corner piece is the same length as a side of the cube, figure 1, it follows that adding 700 to 1400 or 700+700, the sum 2100 will represent the total length of the three *corner pieces*. Were we to multiply 2100 by 20, we should obtain the area of the three *corner pieces*, which might be added to 1470000, the area of the three *slabs*. But, since there is also to be added a *little cube*, each of whose sides is 20 *linear* feet, we will add 20 to 2100, and thus obtain 2120 for the total length of the three corner pieces, and of a side of the little cube. Now, multiplying 2120 by 20, we obtain 42400 *square* feet for the surface of the three corner pieces and a face of the little cube; which, added to 147000, the number of square feet in the faces of the three slabs, will give 1512400 *square* feet in all the additions. If we multiply 1512400 by 20, the thickness of these additions, we shall obtain 30248000 *cubic* feet for all the additions, which, subtracted from 39657176, leaves 9409176 *cubic* feet. The cube thus completed is 720 feet on a side, and is represented by figure 4.

Fig. 3.

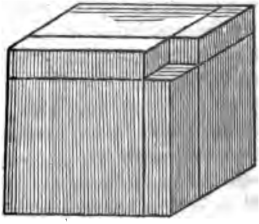
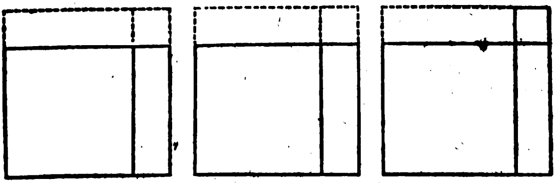


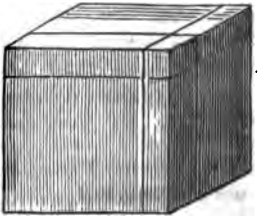
Figure a.



The *surfaces* now obtained may be represented (figure a,) by the parts included within the *heavy* lines. The three divisions of the figure, including the *dotted* lines, may be supposed to be three *entire* faces of the cube, figure 4.

But this cube is to be further increased by 9409176 *cubic* feet. And as before, the parts added will consist of three equal *slabs*, three equal *corner pieces*, and a *little cube*. The trial divisor, which is the area of the faces of the three slabs, is the same as three times the area of a face of the cube, figure 4, each of whose sides is 720 feet.

Fig. 4.



Now to obtain this area, we have only to add to the surfaces already obtained, and represented within the heavy lines, (figure a,) three rectangles, each 700 feet by 20, and two little squares 20 feet by 20 feet.

If to 2120, a number which we already have, we add 20, we shall obtain 2140, the *linear* extent of the rectangles and squares desired, as in the dotted portions, (figure a.) And as these dotted portions have all the same width of 20 feet, if we multiply 2140 by 20, we shall obtain 42800 *square* feet for the area of the dotted portion, (figure a,) which, added to 1512400, the area of the parts included within the heavy lines, will give 1555200 *square* feet for the area of three slabs, each equal to one face of the cube, (figure 4.) This will be a second *trial divisor*. We find this divisor contained between 6 and 7 times in 9409176; hence our third figure *of the root*, or the figure in the units' place, is 6. Were we to multiply 1555200 by 6, it would give the cubic feet in the second set of *slabs.* But before multiplying, we will increase that sum by the surface of the second set of *corner pieces*, and of the second *little cube.* The length of each corner piece is the same as a side of the cube, figure 4, which is 720 feet; hence, adding 20 to 2140 already found, we obtain 2160, which, being 3 times 720, will be the linear extent of the three corner pieces. Were we to multiply 2160 by 6, we should find the surface of these three corner pieces, but as we wish also the area of one of the faces of the second *little cube*, we add 6 to 2160, and thus obtain 2166, which, multiplied by 6, will give 12996 for surface of second set of *corner pieces*, and of second *little cube;* this added to 1555200, gives 1568196 for the surface of the whole second series of additions. Multiplying 1568196 by 6, we obtain 9409176 *cubic* feet, which have thus been added to the cube represented by figure 4; hence the cube whose side is 726 feet is the one sought. The above work may be arranged as follows:

1st COLUMN. *Linear feet.*	2d COLUMN. *Square feet.*	NUMBER. *Cubic feet.*	ROOT. *Linear feet.*
700	490000	382657176	(700+20+6=726
1400	1470000=1st tr. divisor,	343000000	
2100	1512400	39657176	
2120	1555200=2d tr. divisor,	30248000	
2140	1568196	9409176	
2160		9409176	
2166		0	

If we omit the ciphers on the right, and omit unnecessary terms, the work will take the following condensed form:

1st COLUMN. *Linear feet.*	2d COLUMN. *Square feet.*	NUMBER. *Cubic feet.*	ROOT. *Linear feet.*
7	49	382657176	(726.
14	147=1st trial divisor,	346	
212	15124	39657	
214	15552=2d trial divisor,	30248	
2166	1568196	9409176	
		9409176	
		0	

NOTE.—In the extraction of the cube root, as just illustrated, it will be noticed that each divisor is a geometrical surface; that is to say, the product of two dimensions, width and breadth, for example; and of course the quotient must be the other dimension, that is, the thickness.

But it is important to remember that it is only squares and cubes, square roots and cube roots, that can have any relation to *geometrical* dimensions; any higher power of a number as 4^5, or any other root as $\sqrt[5]{}$, cannot be illustrated by blocks. The *principle*, therefore, of *involution* and *evolution* is, strictly speaking, independent of surfaces and solids; it is purely *arithmetical.*

From the foregoing demonstration we may deduce the following

RULE.

I. Separate the number whose root is to be found, into periods of three figures each, counting from the units' place towards the left. When the number of figures is not divisible by 3, *the left-hand period will contain less than* 3 *figures.*

II. Seek the greatest figure whose cube shall not exceed the first or left-hand period; write it after the manner of a quotient in division for the first figure of the root. Place this figure for the head of a first left-hand column, and its square for the head of a second left-hand column, and subtract its cube from the first period. To the remainder bring down a second period for the FIRST DIVIDEND. *Add the figure in the root to the term of the* 1ST COLUMN *already found, for its next term, which multiply by the same figure, and add the product to the term already found in the* 2D COLUMN, *for its next term, which will be a* TRIAL DIVISOR.

III. Find how many times the trial divisor, with two ciphers annexed, is contained in the dividend; write the quotient for the next figure of the root. Annex this figure to the last term of the 1ST COLUMN, *after having added to that* *term the preceding quotient figure; this will give the next term of the* 1ST COLUMN. *Multiply this term by the last found figure in the root, and add the product, after advancing it two places to the right, to the last term of the* 2D COLUMN, *for its next term. Multiply this term by the last found figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a* NEW DIVIDEND.

Proceed as before until all the periods have been brought down.

NOTE 1.—When any dividend is not so great as the corresponding trial divisor with two ciphers annexed, write 0 for the next figure of the root, and to the dividend bring down the next period. Use the same trial divisor as before, but with *four* ciphers annexed.

NOTE 2.—The trial divisor, being less than the true divisor, will sometimes give too large a quotient figure; when the multiplication of the true divisor by this figure shows such to be the case, this figure must be made smaller.

NOTE 3.—By the above rule, which is different from the rule usually given by the aid of geometrical diagrams, we have managed to keep distinct all the geometrical magnitudes; thus our first column represents the numerical values of lines, the second column represents the numerical values of surfaces, and the third column corresponds to solids. And, as we are never required to multiply by any number greater than is expressed by a single digit, the labor of multiplying and adding results to the terms of the successive columns is far simpler than at first might be supposed.

By means of these auxiliary columns the work bears a close analogy to Horner's method of solving numerical cubic equations. (See Treatise on Algebra.) The use of auxiliary columns becomes very apparent in the extraction of roots of the higher orders, as the fifth root, the seventh root, &c.

EXAMPLES.

What is the cube root of 913517247483640899?

OPERATION.

1st Column.	2d Column.	Number.	Root.
		913'517'247'483'640'899(	970299
9	81	729	
18	243	184517	
277	26239	183673	
284	28227	844247483	
29102	282328204	564656408	
29104	282386412	279591075640	
291069	28241260821	254171347389	
291078	28243880523	25419728251899	
2910879	2824414250211	25419728251899	
		0	

2. What is the cube root of 10077696? *Ans.* 216.

3. What is the cube root of 2357947691? *Ans.* 1331.

4. What is the cube root of 42875? *Ans.* 35.

5. What is the cube root of 117649? *Ans.* 49.

6. What is the cube root of 7256313856? *Ans.* 1936.

CASE II.

To extract the cube root of a decimal fraction, or of a number consisting partly of a whole number and partly of a decimal, we have this

RULE.

I. Annex ciphers to the decimals, if necessary, so that they may be separated into equal periods.

II. Separate the decimals into periods of 3 figures each, counting from the decimal point toward the right, and proceed as in whole numbers.

NOTE.—If the given number has not an exact root, there will be a remainder after all the periods have been brought down. The process may be continued by annexing ciphers for new periods.

EXAMPLES.

1. What is the cube root of 0·469640998917? *Ans.* 0·7773.
2. What is the cube root of 18·609625? *Ans.* 2·65.
3. What is the cube root of 1·25992105? *Ans.* 1·08005.
4. What is the cube root of 2? *Ans.* 1·2599.
5. What is the cube root of 9? *Ans.* 2·08008.
6. What is the cube root of 3? *Ans.* 1·4422.

CASE III.

To extract the cube root of a vulgar fraction, or mixed number, we have this

RULE.

I. Reduce the fraction or mixed number, to its simplest fractional form.

II. Extract the cube root of the numerator and denominator separately, if they have exact roots, but when they have not, reduce the fraction to a decimal, and then extract the root by Case II.

EXAMPLES.

1. What is the cube root of $\frac{2197}{4913}$? *Ans.* $\frac{13}{17}$.
2. What is the cube root of $\frac{85162}{170723}$? *Ans.* $\frac{23}{29}$.
3. What is the cube root of $17\frac{1}{8}$? *Ans.* 2·577, nearly.
4. What is the cube root of $5\frac{1}{7}$? *Ans.* 1·726, nearly.
5. What is the cube root of $\frac{81}{99}$? *Ans.* 0·9353, nearly.
6. What is the cube root of $\frac{2}{3}$? *Ans.* 0·8736, nearly.

EXAMPLES INVOLVING THE PRINCIPLES OF THE CUBE ROOT.

136. *It is an established theorem of geometry, that all similar solids are to each other as the cubes of their like dimensions.*

1. If a cannon-ball, 3 inches in diameter, weigh 8 pounds, what will a ball of the same metal weigh, whose diameter is 4 inches?

By the above theorem, we have

$$3^3 : 4^3 :: 8 \text{ pounds} : 18\tfrac{26}{27} \text{ pounds},$$

for the answer.

2. The celebrated Stockton gun, which, in bursting, proved so fatal to many of our distinguished citizens, is said to have carried a ball 12 inches in diameter, which weighed 238 pounds. What ought to be the diameter of another ball of the same metal, which should weigh 32 pounds?

$\frac{32}{238} \times 12^3 = 232{\cdot}336$ inches nearly $=$ cube of the diameter of the ball sought.

Hence, $\sqrt[3]{232 \cdot 336} = 6 \cdot 1476$ inches nearly, the diameter of the ball required.

3. A cooper having a cask 40 inches long and 32 inches at the bung diameter, wishes to make another cask of the same shape, which shall contain just twice as much. What will be the dimensions of the new cask?

Ans. $\begin{cases} 40\sqrt[3]{2} = 50 \cdot 3968 \text{ inches, nearly, for length.} \\ 32\sqrt[3]{2} = 40 \cdot 3175 \text{ inches, nearly, for diameter.} \end{cases}$

4. What is the side of a cube, which will contain as much as a chest 8 feet 3 inches long, 3 feet wide, and 2 feet 7 inches deep? *Ans.* 47·984 inches, nearly.

5. How many cubic quarter inches can be made out of a cubic inch? *Ans.* 64.

6. Required the dimensions of a rectangular box, which shall contain 20000 solid inches, the length, breadth, and depth being to each other, as 4, 3, and 2.

SOLUTION.

$\frac{20000}{1} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{2500}{3}$, whose cube root is $5\sqrt[3]{\frac{20}{3}} =$ 9·4103, nearly.

$$Ans. \begin{cases} 9 \cdot 4103 \times 4 = 37 \cdot 6412, \text{ length.} \\ 9 \cdot 4103 \times 3 = 28 \cdot 2309, \text{ breadth.} \\ 9 \cdot 4103 \times 2 = 18 \cdot 8206, \text{ depth.} \end{cases}$$

Or, as follows:

If we were to augment the width of this box, so as to make it as wide as it is long, its volume would become $\frac{4}{3}$ of $20000 = 26666\frac{2}{3}$. Again, if we augment the depth of this new box, so that it may be as deep as it is wide, and as it is long, its volume will become 2 times $26666\frac{2}{3} = 53333\frac{1}{3}$, which is the contents of a cubical box, whose side is equal to the length of the original box. Hence,

$\sqrt[3]{53333\frac{1}{3}}=37{\cdot}641$, nearly, for the length. The width is $\frac{3}{4}$ of this length, and the depth is $\frac{1}{2}$ this length.

Note.—For a more complete treatise on the square and cube root, as well as the roots of all powers, see Higher Arithmetic.

ARITHMETICAL PROGRESSION.

137. A series of numbers, which succeed each other regularly, by a common difference, is said to be in *arithmetical progression.*

When the terms are constantly increasing, the series is an *arithmetical progression ascending.*

When the terms are constantly decreasing, the series is an *arithmetical progression descending.*

Thus, 1, 3, 5, 7, 9, &c., is an ascending arithmetical progression; and 10, 8, 6, 4, 2, is a descending arithmetical progression.

The terms of an arithmetical progression may be fractional. Thus, in the progressions,

$\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c.;
$\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3, &c.

The first has a common difference of $\frac{1}{2}$; the second has a common difference of $\frac{1}{3}$.

In arithmetical progression, there are five things to be considered:

1. *The first term.*
2. *The last term.*

3 *The common difference.*
4 *The number of terms.*
5. *The sum of all the terms.*

These quantities are so related to each other, that any three of them being given, the remaining two can be found.

We will demonstrate one or two of the most important cases.

When are numbers in arithmetical progression? When is the progression ascending? When is it descending? Are the numbers 1, 3, 5, 7, 9, &c., in ascending or descending arithmetical progression? Mention the five quantities to be considered in arithmetical progression. How many of these must be given in order to be able to find the others?

CASE I.

By our definition of an ascending arithmetical progression, it follows that the second term is equal to the first, increased by the common difference; the third is equal to the first, increased by twice the common difference; the fourth is equal to the first, increased by three times the common difference; and so on, for the succeeding term.

Hence, when we have given the first term, the common difference, and the number of terms, to find the last term, we have this

RULE.

To the first term add the product of the common difference into the number of terms, less one.

EXAMPLES.

1. What is the 100th term of an arithmetical progression, whose first term is 2, and common difference 3?

In this example, the number of terms, less one, is 99, which, multiplied by the common difference, 3, gives 297, which, added to the first term, 2, makes 299 for the 100th term.

2. What is the 50th term of the arithmetical progression, whose first term is 1, the common difference $\frac{1}{2}$? *Ans.* $25\frac{1}{2}$.

3. A man buys 10 sheep, giving $1 for the first, $3 for the second, $5 for the third, and so on, increasing in arithmetical progression. What did the last sheep cost at that rate? *Ans.* $19.

4. The first term of an arithmetical progression is $\frac{3}{4}$ the common difference $\frac{1}{8}$, and the number of terms 26. What is the last term? *Ans.* $3\frac{7}{8}$.

CASE II.

From the nature of an arithmetical progression, we see that the second term added to the next to the last term is equal to the first added to the last; since the second term is as much greater than the first, as the next to the last is less than the last. After the same method of reasoning, we infer that the sum of any two terms equidistant from the extremes, is equal to the sum of the extremes.

Hence, it follows that the terms will average just half the sum of the extremes.

Therefore, when we have given the first term, the last term, and the number of terms, to find the sum of all the *terms*, we have this

RULE.

Multiply half the sum of the extremes by the number of terms.

EXAMPLES.

1. The first term of an arithmetical progression is 2, the last term is 50, and the number of terms is 17. What is the sum of all the terms?

In this example, half the sum of the extremes is

$$\tfrac{1}{2} \text{ of } (2+50)=26.$$

This, multiplied by the number of terms, gives $26\times17=442$, for the sum required.

2. The first term of an arithmetical progression is 13, the last term is 1003, the number of terms is 100. What is the sum of the progression? *Ans.* 50800.

3. A person travels 25 days, going 11 miles the first day, 135 the last day; the miles which he traveled in the successive days, form an arithmetical progression. How far did he go in the 25 days? *Ans.* 1825 miles.

4. Bought 7 books, the prices of which are in arithmetical progression. The price of the first was 8 shillings, and the price of the last was 28 shillings. What did they all come to? *Ans.* £6 6*s*.

5. What is the sum of 1000 terms of an arithmetical progression, whose first term is 7 and last term 1113? *Ans.* 560000.

6. The first term of an arithmetical progression is $\frac{3}{7}$, and the last term $365\frac{4}{7}$, and the number of terms 799. What is the sum of all the terms? *Ans.* 146217.

GEOMETRICAL PROGRESSION

138. A SERIES of numbers which succeed each other regularly, by a constant multiplier, is called a *geometrical progression.*

This constant factor, by which the successive terms are multiplied, is called the *ratio.*

When the ratio is greater than a unit, the series is called an *ascending geometrical progression.*

When the ratio is less than a unit, the series is called a *descending geometrical progression.*

Thus, 1, 3, 9, 27, 81, &c., is an ascending geometrical progression, whose ratio is 3.

And, 1 $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c., is a descending geometrical progression, whose ratio is $\frac{1}{4}$.

In geometrical progression, as in arithmetical progression, there are five things to be considered.

1 *The first term.*
2. *The last term.*
3. *The common ratio.*
4. *The number of terms.*
5. *The sum of all the terms.*

These quantities are so related to each other, that any three being given, the remaining two can be found.

The solution of some of these cases requires a knowledge of higher principles of mathematics than can be detailed by arithmetic alone.

We will give a demonstration of the rules of one or two of the most important cases.

When are numbers in geometrical progression? What is the constant factor, by which the successive terms are multiplied, called? When this ratio exceeds a unit, the progression is called what? When this ratio is less than a unit, how is the progression called? Give an example of an ascending geometrical progression? Give an example of a descending geometrical progression. How many quantities are to be considered in geometrical progression? Mention these quantities. How many of these must be known to enable us to find the others?

CASE I.

By the definition of a geometrical progression, it follows that the second term is equal to the first term, multiplied by the ratio; the third term is equal to the first term, multiplied by the second power of the ratio; the fourth term is equal to the first term, multiplied by the third power of the ratio; and so on, for the succeeding terms.

Hence, when we have given the first term, the ratio, and the number of terms, to find the last term, we have this

RULE.

Multiply the first term by the power of the ratio, whose exponent is one less than the number of terms.

EXAMPLES.

1. The first term of a geometrical progression is 1, the ratio is 2, and the number of terms is 7. What is the last term?

In this example, the power of the ratio, whose exponent is one less than the number of terms, is $2^6=64$, which, multiplied by the first term, 1, still remains 64, for the last term.

2. The first term of a geometrical progression is 5, the ratio is 4, and the number of terms 9. What is the last term? Ans. 327680.

3. A person traveling, goes 5 miles the first day, 10 miles the second day, 20 miles the third day, and so on, increasing in geometrical progression. If he continue to travel in this way for 7 days, how far will he go the last day? *Ans.* 320 miles.

CASE II.

Let it be required to find the sum of all the terms of the geometrical progression 2, 6, 18, 54, 162, 486.

If we multiply each term by 3, which is the ratio, we shall obtain this second progression, 6, 18, 54, 162, 486, 1458, the sum of whose terms is obviously 3 times as great as the sum of the terms of the first progression. Consequently, the difference between the sums of the terms of these two progressions is $(3-1)=2$ times the sum of all the terms of the first progression. If we omit the first term of the first progression, it will agree with the second progression, after omitting its last term. Hence, the difference between the sums of the terms of these two progressions may be found by subtracting 2, the first term of the first progression, from 1458, the last term of the second progression; but 1458 was obtained by multiplying 486, the last term of the first progression, by 3, the ratio.

Hence, we finally obtain this condition:

That the sum of all the terms of a geometrical progression, repeated as many times as there are units in the ratio, less one, is equal to the last term multiplied by the ratio, and diminished by the first term.

Therefore, when we have given the first term of a geometrical progression, the last term, and the ratio, to find the sum of all the terms, we have this

RULE.

Subtract the first term from the product of the last term into the ratio; divide the remainder by the ratio, less one.

EXAMPLES.

1. The first term of a geometrical progression is 4, the last term is 78732, and the ratio is 3. What is the sum of all the terms?

In this example, the first term subtracted from the product of the last term into the ratio, is 236192, which, divided by the ratio, less one, gives 118096, for the sum of all the terms.

2. The first term of a geometrical progression is 5, the last term is 327680, and the ratio is 4. What is the sum of all the terms? *Ans.* 436905.

3. A person sowed a peck of wheat, and used the whole crop for seed the following year; the produce of this second year again for seed the third year, and so on. If in the last year, his crop is 1048576 pecks, how many pecks did he raise in all, allowing the increase to have been in a fourfold ratio? *Ans.* 1398101 pecks.

139. When the ratio of a geometrical progression is less than a unit, the first term will be the largest, and the last term the least; the progression will, in this case, be descending; but if we consider the series of terms in a reverse order, that is, calling the last term the first, and the first the last, the progression may then be considered as ascending.

If a decreasing geometrical progression be continued to an infinite number of terms, we may neglect the last term as of no appreciable value; we can find its sum by Case II., when it is modified, as follows:

Given the first term of a descending geometrical progression, and the ratio, to find the sum of all the terms, when continued to infinity.

RULE.

Divide the first term by a unit diminished by the ratio.

EXAMPLE.

1. What is the sum of all the terms of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c.?

In this example, a unit, diminished by the ratio, is $1-\frac{1}{2}=\frac{1}{2}$, and the first term, 1, divided by $\frac{1}{2}$, gives 2, for the sum of all the terms.

2. What is the sum of the infinite series 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c.? *Ans.* $1\frac{1}{2}$.

3. What is the sum of the infinite series $\frac{1}{10}$, $\frac{3}{100}$, $\frac{9}{1000}$, $\frac{27}{10000}$, &c.? *Ans.* $\frac{1}{7}$.

4. What is the sum of the infinite series $\frac{2}{10}$, $\frac{4}{100}$, $\frac{8}{1000}$, $\frac{16}{10000}$, &c.? *Ans.* $\frac{1}{4}$.

ALLIGATION.

140. ALLIGATION is generally treated under two distinct heads, called *Allegation Medial* and *Allegation Alternate.* The latter, however, belongs properly to the province of Algebra.

ALLIGATION MEDIAL.

141. ALLIGATION MEDIAL teaches the method of finding the mean value of a compound, when its several ingredients and their respective values are given.

What is Alligation Medial?

Suppose a grocer mixes 140 pounds of tea, which is worth 8*s.* per pound; 200 pounds, worth 6*s.* per pound; and 160 pounds, worth 10*s.* per pound. What is a pound of the mixture worth?

140 pounds of tea, at 8*s.* per pound, is worth $140\times8=1120s.$; 200 pounds, at 6*s.*, is worth $200\times6=1200s.$; 160 pounds, at 10*s.*, is worth $160\times10=1600s.$ Therefore, the mixture, which is 500 pounds, is worth $1120+1200+1600=3920s.$ Hence, one pound of the mixture must be worth $\frac{3920}{500}=7\frac{21}{25}s.$

Hence, to find the mean value of a compound, composed of several ingredients of different values, we have this

RULE.

Divide the sum of the values of all the ingredients by the sum of the ingredients.

Repeat this Rule.

EXAMPLES.

1. A wine-merchant mixed several sorts of wine, viz.: 32 gallons, at 40 cents per gallon; 15 gallons, at 60 cents per gallon; 45 gallons, at 48 cents per gallon; and 8 gallons, at 85 cents per gallon. What is the value of a gallon of the mixture?

32 gallons, at 40 cents = \$12·80
15 " 60 " = 9·00
45 " 48 " = 21·60
8 " 85 " = 6·80

100 gallons of mixture = \$50·20.

Therefore, one gallon of the mixture is worth \$50·20 ÷ 100 = \$0·502 = 50 cents and 2 mills.

2. A farmer mixed together 7 bushels of rye, worth 72 cents per bushel; 15 bushels of corn, worth 60 cents per bushel; and 12 bushels of wheat, worth \$1·20 per bushel. What is the value of a bushel of the mixture?

Ans. \$0·83$\frac{11}{17}$.

3. A goldsmith melts together 11 ounces of gold 23 carats fine, 8 ounces 21 carats fine, 10 ounces of pure gold, and 2 pounds of alloy. How many carats fine is the mixture? *Ans.* 12$\frac{25}{53}$.

It will be understood that a *carat* is a 24th part. Thus, 21 carats fine is the same as $\frac{21}{24}$ pure metal; in the same way, 23 carats fine is $\frac{23}{24}$ pure metal.

4. On a certain day, the mercury in the thermometer was observed to stand 2 hours at 62 degrees, 4 hours at 70 degrees, 5 hours at 72 degrees, 3 hours at 59 degrees, and 1 hour at 75 degrees. What was the mean temperature for the fifteen hours? *Ans.* 67$\frac{11}{15}$ degrees.

5. Suppose a ship sail at the rate of 5 knots for 3 hours, at 7 knots for 5 hours, and 8 knots for 4 hours. What is her rate of sailing during the 12 hours?

Ans. 6$\frac{5}{6}$ knots.

6. A grocer mixes 30 pounds of sugar worth 10 cents per pound; 40 pounds worth 10$\frac{1}{4}$ cents per pound; 24 pounds worth 11 cents per pound; and 60 pounds worth 13 cents per pound. What is a pound of the mixture worth?

Ans. 11$\frac{30}{77}$ cents.

ALLIGATION ALTERNATE.

142. ALLIGATION ALTERNATE is the reverse of Alligation Medial; that is, it teaches the method of finding the ingredients when their rates are given, so that the compound shall have a given value.

What is Alligation Alternate?

Suppose we wish to mix teas, which are worth 4 and 6 shillings per pound, so that the mixture may be worth 5 shillings per pound: it is obvious that we must take equal quantities of each; since the price of the one is as much less than the mean price, as the other is greater.

Again, suppose we wish to mix teas, which are worth 4 and 7 shillings per pound, so that the mixture may be worth 5 shillings. In this case the 7 shilling tea is 2 shillings above the average price, whilst the 4 shilling tea is but 1 shilling below: it will be necessary to use twice as much of the 4 shilling tea as of the 7 shilling tea; and in all cases it is obvious that the quantities to be used will be in the inverse ratio to the differences between their prices and the mean price.

When there are more than two simples they may be compared together in couplets, one term of which must obviously exceed the average price, while the other must be less.

CASE I.

The rates of the several ingredients being given, to make a compound of a fixed rate.

From what has been said above, we draw the following

RULE.

I. Write the rates of the simples in a column under each other, then connect each rate of the ingredients which is less than the rate of the compound, with one or more rates greater than the rate of the compound; connect in the same way, each rate which is greater than the rate of the compound, with one or more rates which are less.

II. Write the difference between each rate of the ingredients and the compound rate, opposite the rate of the ingredients with which it is connected. If only one difference stands against any rate, it will be the required quantity of the ingredient of that rate; but if there be several, their sum will be the quantity required.

Repeat this Rule.

EXAMPLES.

1. How much sugar at 5, 6, and 10 cents per pound, must be mixed together, so that a pound of the mixture may be worth 8 cents?

SOLUTION.

$$8\left\{\begin{array}{l}5\\6\\10\end{array}\right. \qquad \begin{array}{r}2\\2\\3+2=5\end{array}$$

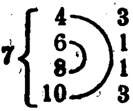
Therefore, if we take 2 pounds at 5 cents, 2 pounds at 6 cents, and 5 pounds at 10 cents, we shall satisfy the conditions of the question. It is obvious, that any other number of pounds which are to each other as the numbers

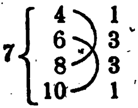
2, 2, and 5, will satisfy the question equally well; so that in Alligation Alternate the number of solutions are *indefinite*; all that we can do is to find the ratios of the quantities required.

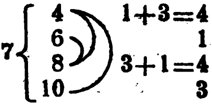
NOTE.—In many cases the ingredients will admit of being connected in several ways, and then we shall obtain as many sets of ratios as there are methods of connecting them.

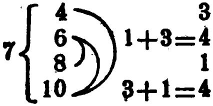
2. How many pounds of raisins at 4, 6, 8, and 10 cents per pound, must be mixed, so that a pound of the compound may be worth 7 cents?

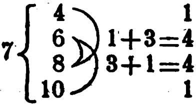
In this question, the terms may be connected in seven distinct ways; therefore, we shall obtain seven sets of ratios, as follows:

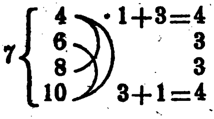


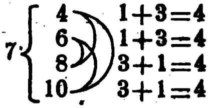












3. How much wine, at 72 cents per gallon, and 48 cents per gallon, must be mixed together, that the composition may be worth 60 cents per gallon?

Ans. An equal quantity of each.

4. How many gallons of wine and water must be mixed together, so that the mixture may be worth 60 cents per

gallon, the water being considered of no value, and the wine with which it is mixed being worth 90 cents per gallon? *Ans*. 2 gallons of wine to 1 of water.

5. Having gold of 12, 16, 17, and 22 carats fine, what proportion of each kind must I take, to make a compound of 18 carats fine? *Ans*. 4, 4, 4, 9.

6. It is required to mix different sorts of grain, at 56, 62, and 75 cents per bushel, so that the mixture may be worth 60 cents per bushel. How much of each kind must be taken? *Ans*. 17, 4, 4.

Besides the variety of answers which may be obtained by connecting the simples differently, an infinite number of solutions may be found, by combining the different ratios, as we will illustrate by the aid of the following question:

7. How much tea at 5 shillings, 6 shillings and 8 shillings per pound, must be mixed so that the mixture may be worth 7 shillings per pound?

If we compound only the 5 and 8 shilling teas, we must take them in the ratio of 1 to 2, since 7 shillings is 1 shilling less than 8 shillings, and 2 shillings greater than 5 shillings. Hence, any one of the compounds in the following group (A,) will be worth 7 shillings per pound.

	(1)	(2)	(3)	(4)	(5)	(6)	
5 shilling tea	1	2	3	4	5	6, &c.	(A.)
8 shilling tea	2	4	6	8	10	12, &c.	
Sums,	3;	6;	9;	12;	15;	18; &c.	

If we now mix the 6 and 8 shilling teas, we see that it will be necessary to take equal quantities of each, since the average price is to be as much above 6 shillings as it is below 8 shillings. Hence, the following compound will *also* be worth 7 shillings per pound:

	(1)	(2)	(3)	(4)	(5)	(6)		
6 shilling tea	1	2	3	4	5	6	&c.	(B)
8 shilling tea	1	2	3	4	5	6	&c.	
Sums,	2;	4;	6;	8;	10;	12;	&c.	

Now, it is obvious, we may combine any one of these last results with any one of the former results. Thus, if we combine (1) of group (A) with (1) of (B) we have

	Pounds.
5 shilling tea . . .	1
6 " " . . .	1
8 " "	$2+1=3$

If we combine (1) of (A,) with (2) of (B,) we have,

	Pounds.
5 shilling tea . . .	1
6 " " . . .	2
8 " "	$2+2=4$

Combining (2) of (A,) with (3) of (B,) we have,

	Pounds.
5 shilling tea . . .	2
6 " " . . .	3
8 " "	$4+3=7$

Combining (5) of (A,) with (4) of (B,) we have,

	Pounds.
5 shilling tea . . .	5
6 " " . . .	4
8 " "	$10+4=14$

The number of combinations which could be made in this way is unlimited; hence, the above class of questions in Alligation admit of an infinite number of answers.

CASE II.

When one of the ingredients is limited to a certain *quantity.*

1. A person wishes to mix 10 bushels of wheat, worth $1 per bushel, with rye, worth 70 cents per bushel, and oats worth 30 cents per bushel, so that the mixture may be worth 60 cents per bushel. How many bushels of rye and oats must he use?

Proceeding, according to Case I., we find the proportionate numbers to be 30, 30, and 50. Hence,

30 : 30 : : 10 : 10
30 : 50 : : 10 : 16⅔.

So that he must make use of 10 bushels of rye, and 16⅔ bushels of oats. Hence, this

RULE.

Find the proportionate quantities of each ingredient, by Case I., in the same manner as though there was no limitation; then, as the difference against the simple whose quantity is given, is to each of the other differences, so is the given quantity of that simple to the quantity required of each of the other simples.

Repeat this Rule.

2. A grocer has 90 pounds of tea, worth 90 cents per pound, which he wishes to mix with three other qualities, valued at 80 cents, 70 cents, and 60 cents per pound. How much must he take of these three kinds, so as to be able to sell the mixture at 85 cents per pound?

Ans. 10 pounds of each.

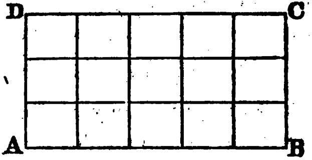
3. A merchant has 90 pounds of spice worth 86 cents per pound, which he wishes to mix with three other sorts which are worth 30, 40, and 50 cents per pound, respectively. How many pounds must be used so that the compound may be worth 55 cents per pound?

Ans. He must use 62 pounds of each.

MENSURATION.

143. For the reason of many of the rules which we shall give for measuring surfaces and solids, we shall refer to the principles of geometry. The reference being in all cases to the "Elements of Geometry."

Problem I.—*To find the area of a rectangle.*



Suppose A B C D to be a rectangle whose length is 5 feet, and width 3 feet.

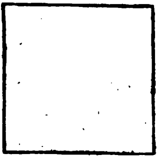
If we divide this rectangle into portions of one square foot each, by means of lines drawn parallel to the sides of the rectangle, we shall obtain 15 such squares; that is, the rectangle will contain 15 square feet. In this example there are 3 strips of 5 square feet in each, or 5 strips of 3 square feet each. So that the number of square feet is found by multiplying the number of feet in length by the number of feet in width.

Hence, to find the area of a rectangle we have this

RULE.

Multiply the length by the width, and the product will denote the number of squares of the same kind as the measure used in estimating the sides of the rectangle. If the sides of the rectangle are measured in feet, the product will be the *square feet; if in inches, then the product will be square inches*, (B. IV. Prop. II, Scholium.)

Note.—When the width of the rectangle is the same as its length, it becomes a square, in which case we multiply the side of the square into itself.



EXAMPLES.

1. How many square feet in a floor which is 16 feet wide and $23\frac{1}{2}$ feet long? And how many yards of carpeting, one yard wide, will cover the floor?

$23\frac{1}{2} \times 16 = 376 =$ the number of square feet.

Since in one square yard there are 9 square feet, we find $376 \div 9 = 41\frac{7}{9} =$ the number of yards of carpeting required.

2. In a table 5 feet 3 inches long, and 3 feet 2 inches wide, how many square inches? And how many square feet?

Ans. 2394 sq. inches.
$16\frac{5}{8}$ sq. feet.

3. In a rectangular field which is 13 rods long, and 7 rods wide, how many square rods? And what part is it of an acre?

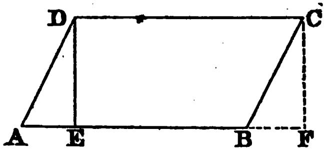
Ans. 91 sq. rods.
$\frac{91}{160}$ of an acre.

4. How many square inches in a square board $10\frac{1}{2}$ inches on a side? *Ans.* $110\frac{1}{4}$ sq. inches.

5. Which is the greater, a square board of 9 inches on a side, or a rectangular one 12 inches long and $7\frac{1}{2}$ wide?

Ans. The rectangular piece contains 9 square inches more than the square one.

PROBLEM II.—*To find the area of a parallelogram.*



Let A B C D be a parallelogram having A B for its base and D E its altitude. If from C we draw C F perpendicular to the base A B, meeting it, produced at the point F, the figure E F C D will be a rectangle equivalent to the parallelogram, since the triangle A E D is obviously equal to the triangle B F C. The base E F of the rectangle is equal to A B, the base of the parallelogram. The area of the rectangle is found (PROB. I.) by multiplying the base by its altitude, and since the parallelogram is equal to the rectangle, and since its base and altitude are respectively equal to the base and altitude of the rectangle, it follows that the area of the parallelogram may be found by multiplying its base by its altitude.

Hence, to find the area of a parallelogram, we have this

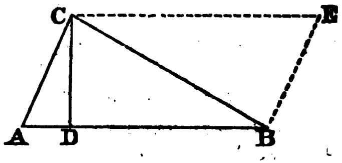
RULE.

Multiply the base by the altitude.

NOTE.—This rule includes the rule under the last problem for finding the area of a rectangle or square. It is not therefore necessary to add any new examples under this problem.

PROBLEM III.—*To find the area of a triangle.*

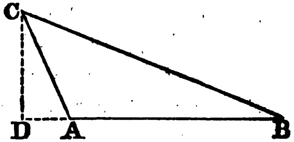
Let A B C be a triangle, having A B for its base and C D its altitude. By drawing C E parallel to the base



A B, and B E parallel to the side, A C, we shall form a parallelogram A B E C, evidently double the triangle A B C. The area of the parallelogram is found (PROB. II,) by multiplying the base A B into the altitude C D. And as the triangle is one half the parallelogram, its area may be found by this

RULE.

Multiply half the base by the altitude.



NOTE.—Either side of the triangle may be regarded as the base, and the altitude will be the perpendicular drawn from the opposite angle to the base, or to the base produced. In the annexed diagram, the perpendicular meets the base produced. The above rule applies equally well in this case, the area being found by multiplying half the base A B into C D.

When the three sides of a triangle are known, the area may be found by this second

RULE.

From the half sum of the three sides, subtract separately each side, take the square root of the continued product of the three remainders and half sum, and it will give the area.

NOTE.—For a demonstration of this second rule, see Geometry, *B. II.* Prop. IX.

EXAMPLES.

1. What is the area of a triangle whose base is 12 feet, and altitude 3 yards?

3 yards = 9 feet. Therefore $\frac{1}{2}$ of $12\times9=54$ square feet, or 6 square yards for the area.

2. What is the area of a triangle whose sides are respectively 7, 11 and 12 feet?

SOLUTION.

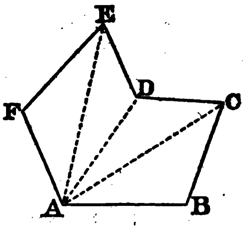
$\frac{1}{2}$ of $(7+11+12)=15$ $\quad 15\times8\times4\times3=1440.$
$15-7=8$
$15-11=4$ $\quad \sqrt{1440}=12\sqrt{10}=37{\cdot}95$ nearly.
$15-12=3$ $\quad$ Hence the area is 37·95 sq. feet.

3. What is the area of a triangle whose base is 14 rods, and whose altitude is 12 rods? *Ans.* 84 sq. rods.

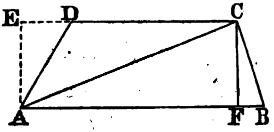
4. What is the area of a triangle whose sides are respectively 13, 14 and 15 yards? *Ans.* 84 sq. yards.

5. In a triangular field whose sides are 18, 80 and 82 feet, how many square yards? *Ans.* 80 sq. yards.

The area of any figure which is limited by any number of right lines, as the field A B C D E F, may be found by dividing it into triangles, and then computing each triangle separately, and taking their sum.



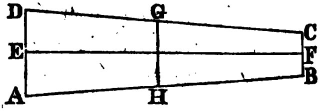
Problem IV.—*To find the area of a trapezoid.*



Let ABCD be a trapezoid having AB and CD for the parallel sides, CF for its altitude. If we draw AC it will divide the trapezoid into two triangles ABC, CDA. The area of the triangle ABC may be found (Prop. III.) by multiplying half the base AB into the altitude CF; and the area of the triangle CDA is found by multiplying half the base CD into the altitude AE, or into its equal CF. Hence the area of the trapezoid, which is the sum of the two-triangles, may be found by the following

RULE.

Multiply half the sum of the two parallel sides by the altitude.



This rule has a fine application in measuring a tapering board, as ABCD. In this case half the sum of the parallel sides, AD and BC, is found by measuring the width GH at the middle of the board. This average width, GH, being multiplied by the length EF will give its area.

EXAMPLES.

1. If the parallel sides of a trapezoidal garden are respectively 4 and 6 rods; and the perpendicular distance

between these sides is 8 rods, how many square rods in the garden?

Ans. { 40 sq. rods, or just $\frac{1}{4}$ of an acre.

2. How many square feet in a tapering board 16 feet long, measuring 15 inches wide at one end, and 10 inches at the other? *Ans.* $16\frac{2}{3}$ sq. feet.

PROBLEM V.—*The diameter of a circle being given to find its circumference.*

If the diameter of a circle is taken as a unit, the circumference will be 3·14159265, nearly. The exact value of the ratio of the circumference to the diameter has never been found. Its approximate value has been extended to more than 200 places of decimals. (Geometry, B. V, Prop. XIV, Scholium.)

Hence, when the diameter of a circle is known, its circumference may be found by the following

RULE.

Multiply the diameter by 3·1416.

NOTE.—In the Higher Arithmetic, under Continued Fractions, we found some of the approximate values of this ratio to be 3, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$, &c. This last value of $\frac{355}{113}$ is true to six places of decimals. It may be easily retained in the memory by observing that if the first three odd numbers, 1, 3, 5, be duplicated, they will stand 113355. Now the first three figures give the denominator, and the other three give the numerator of the ratio.

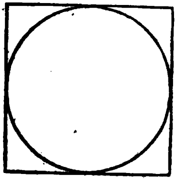
EXAMPLE.—What is the circumference of the earth, on the supposition that it is 8000 miles in diameter?

Ans. $3·1416 \times 8000 = 25132·8$ miles, nearly.

Problem VI.—*To find the area of a circle, when its diameter is known.*

RULE.

Multiply the circumference by one fourth of the diameter. Or, what is equivalent, multiply the square of the diameter by 0·7854=¼ *of* 3·1416. (Geometry, B. V., Prop. XI.)



Note.—If a circle be inscribed in a square, its area will be to the area of the square, as 0·7854 is to 1.

EXAMPLES.

1. How many acres in a circle one mile in diameter?

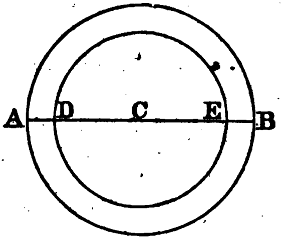
In a square mile there are 640 acres, therefore in a circle one mile in diameter there are

640 acres × 0·7854 = 502·656 acres.

2. Which is the greater area, a circle 5 feet in diameter, or the sum of the areas of two other circles, the one being 4 feet in diameter and the other 3 feet?

Ans. The first circle is equal in area to the sum of the other two.

From the above rule we may deduce a simple method of finding the area comprised between the circumferences of two concentric circles, which area is the difference between two circles.



The area of the circle whose diameter is A B, is found by multiplying its square by 0·7854. And the circle whose diameter is D E, is found by multiplying the square of this diameter by 0·7854. Hence, the difference of these areas is equal to the difference of the squares of the diameters multiplied by 0·7854.

Problem VII.—*To find the solidity of a prism, or of a cylinder.*

RULE.

Multiply the area of the base by the altitude. (Geometry, B. VII., Prop. XI.)

EXAMPLES.

1. How many cubic feet in a rectangular stick of timber 10 inches by 12 inches, and 36 feet long?

10 inches $=\frac{5}{6}$ of a foot, which is the fractional part of a square foot for the area of the end.

$\frac{5}{6}\times 36=30=$ number of cubic feet.

2. In a cylindrical log 14 feet long, and 14 inches in diameter, how many cubic feet?

14 inches $=1\frac{1}{6}$ feet $=\frac{7}{6}$ of a foot.

$\frac{7}{6}\times\frac{7}{6}\times 0{\cdot}7854=1{\cdot}069$ square feet for area of end.

$1{\cdot}069\times 14=14{\cdot}966$ cubic feet.

3. How many cubic inches in a round bar of iron, 20 feet long and $\frac{3}{4}$ of an inch in diameter?

Ans. 106·029 cubic inches.

PROBLEM VIII.—*To find the volume of a pyramid, or of a cone.*

RULE.

Multiply the area of the base by one-third the altitude. (Geometry, B. VII., Prop. XVII.; and B. VIII. Prop. V.)

EXAMPLES.

1. The Egyptian pyramid, Cheops, covers a square of $763\frac{2}{5}$ feet on a side, and is 480 feet perpendicular height. How many cubic feet does it contain?

Ans. $93244729\frac{4}{5}$ cubic feet.

2. Suppose the mast of a ship to be a regular cone 87 feet long, and 2 feet in diameter at its base, how many cubic feet will it contain? *Ans.* 91·1064 cubic feet.

PROBLEM IX.—*To find the surface of a sphere, when its diameter is given.*

RULE.

Multiply the square of the diameter by 3·1416. (Geometry, B. VIII., Prop. XIII. Schol.)

EXAMPLES.

1. How many square miles on the surface of the earth, on the supposition that it is an exact sphere of 8000 miles in diameter?

Ans. $8000 \times 8000 \times 3{\cdot}1416 = 201062400$ square miles.

In order to obtain a value true to a unit, we must use, for our multiplier, 3·14159265, instead of 3·1416.

Using this more accurate value, we find the

Ans. 201061930 square miles, nearly.

2. How many superficial inches has a ball 6 inches in diameter? *Ans.* 113·0976 square inches.

Problem X.—*To find the volume of a sphere, when its diameter is given.*

RULE.

Multiply the cube of the diameter by 0·5236, *which is* $\frac{1}{6}$ *of* 3·1416. (Geometry, B. VIII., Prop. XIII. Schol.)

1. How many cubic inches in a ball 6 inches in diameter?

Ans. $6 \times 6 \times 6 \times 0{\cdot}5236 = 113{\cdot}0976$ cubic inches.

Note.—Comparing this Example with Example 2, under last Problem, we see that the number of superficial inches and cubic inches are equal in a sphere of 6 inches in diameter.

2. How many cubic inches in a ball of the celebrated Stockton gun, the diameter of which is 12 inches?

Ans. 904·7808 cubic inches.

The following table of multipliers will be found very convenient for solving nearly all problems which can arise in mensuration of circles and spheres.

TABLE OF MULTIPLIERS.

1. Radius of a circle×6·28318531=Circumference.
2. Square of the radius of a circle×3·14159265=Area.
3. Diameter of a circle×3·14159265=Circumference.
4. Square of the diameter of a circle×0·78539816=Area.
5. Circumference of a circle×0·15915494=Radius.
6. Circumference of a circle×0·31830989=Diameter.
7. Square root of area of a circle×0·56418958=Radius.
8. Square root of area of a circle×1·12837917=Diameter.
9. Radius of circle×1·73205081=Side of inscribed equilateral triangle
10. Side of inscribed equilateral triangle×0·57735027=Radius of circle.
11. Radius of a circle×1·41421356=Side of inscribed square.
12. Side of inscribed square×0·70710678=Radius.
13. Square of radius of a sphere×12·56637061=Surface.
14. Cube of radius of a sphere×4·18879020=Volume.
15. Square of diameter of a sphere×3·14159265=Surface.
16. Cube of diameter of a sphere×0·52359878=Volume.
17. Square of circumference of a sphere×0·31830989=Surface.
18. Cube of circumference of a sphere×0·01688686=Volume.
19. Square root of surface of a sphere×0·28209479=Radius.
20. Square root of surface of a sphere×0·56418958=Diameter.
21. Square root of surface of a sphere×1·77245385=Circumference.
22. Cube root of volume of a sphere×0·62035049=Radius.
23. Cube root of volume of a sphere×1·24070098=Diameter.
24. Cube root of volume of a sphere×3·89777707=Circumference.
25. Radius of a sphere×1·15470054=Side of inscribed cube.
26. Side of inscribed cube×0·86602540=Radius.

PROBLEM XI.—*To find the volume of a frustum of a pyramid, or of a cone.*

RULE.

Find a mean proportional between the area of the two bases, to which add the sum of the bases, and multiply the result by one-third the altitude of the frustum.

EXAMPLES.

1. Suppose a cistern in the form of a frustum of a

cone, to be 9 feet deep, having for diameters 8 feet and 6 feet. How many cubic feet will it contain?

$10^2 \times 0{\cdot}7854 = 100 \times 0{\cdot}7854 =$ area of one base.

$6^2 \times 0{\cdot}7854 = 36 \times 0{\cdot}7854 =$ " other "

$60 \times 0{\cdot}7854 =$ $\left\{ \text{mean proportion between bases.} \right.$

$196 \times 0{\cdot}7854 =$ Sum.

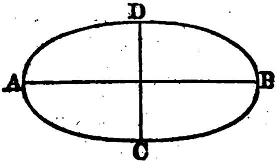
And $196 \times 0{\cdot}7854 \times \frac{1}{3}$ of $9 = 461{\cdot}8152$ cubic feet, for its volume.

2. Suppose a measure to be in the form of a frustum of a regular cone. If its top diameter is 6 inches, and the bottom diameter 9 inches, and it is 12 inches deep, how many cubic inches will it contain? and how many beer gallons of 282 cubic inches each?

Ans. $\left\{ \begin{array}{l} 537{\cdot}2136 \text{ cubic inches.} \\ 1{\cdot}909 \text{ beer gallons.} \end{array} \right.$

Problem XII.—*To find the area of an ellipse.*

Note.—A line drawn through the *centre* of an ellipse is called its diameter. The longest diameter is called the *transverse* diameter; the shortest is called the *conjugate* diameter. Thus AB is the transverse diameter, and CD is the conjugate diameter.



The area of an ellipse may be found by this

RULE.

Multiply the product of the transverse and conjugate diameters by 0·7854.

EXAMPLES.

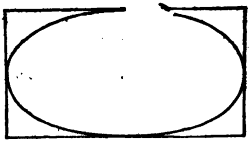
1. How many square feet in the surface of an elliptical pond, whose transverse diameter is 100 feet, and conjugate diameter 60 feet?

Ans. 100 × 60 × 0·7854 = 4712·4 square feet.

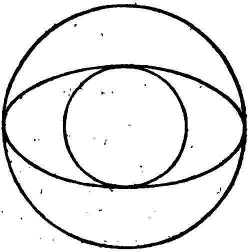
2. How many square inches is an elliptical table whose transverse diameter is 5 feet 3 inches, and conjugate diameter 3 feet 6 inches? And how many square feet?

Ans. { 2078·1684 square inches.
14·4317 square feet.

Note 1.—If an ellipse be inscribed in a rectangle, its area will be to the area of the rectangle as 0·7854 is to 1.



Note 2.—We also infer that, if a circle be inscribed in an ellipse, and another circle be circumscribed about the same ellipse, the ellipse is a mean proportional between the areas of the two circles; that is, we shall have, area of inscribed circle is to the area of ellipse, as area of ellipse is to the area of circumscribed circle.



PROMISCUOUS QUESTIONS.

144. 1. Suppose I purchase $1200 worth of goods, $\frac{1}{3}$ of which is on a credit of 3 months, $\frac{1}{4}$ on a credit of 6 months, and the remaining $\frac{1}{4}$ on a credit of 9 months. How much ready money ought to pay the purchase, interest being 7 per cent.? *Ans.* $1159·64, nearly.

2. In the above example, by the principles of equation of payments, how much credit ought I to have on the whole sum of $1200? *Ans.* 6 months.

3. Now, what is the present worth of $1200 due at the end of 6 months, interest being 7 per cent.?

Ans. $1159·42, nearly.

4. I employed A and B to ditch my meadow. A was to receive $87\frac{1}{2}$ cents per rod, and B was to have $112\frac{1}{2}$ cents per rod; each wrought until his wages amounted to $50. What was the amount of ditch dug by both?

Ans. $101\frac{37}{63}$.

5. Three merchants, A, B, and C, enter into partnership. A advances $1200, B $800, and C $600. A leaves his money 8 months, B 10 months, and C 14 months in the business. They gain $500. What is the share of each?

$$Ans.\begin{cases}\text{A receives } \$184\frac{8}{13}.\\ \text{B} \quad \text{“} \quad 153\frac{11}{13}.\\ \text{C} \quad \text{“} \quad 161\frac{7}{13}.\end{cases}$$

6. A and B have the same income; A saves $\frac{1}{5}$ of his; but B, by spending $120 per annum more than A, at the end of 10 years finds himself $200 in debt. What was the income? *Ans.* $500.

7. Suppose a book to contain 365 pages, averaging 40 lines of 10 words each on each page. How many words would the book contain? *Ans.* 146000 words.

8. There are 31173 verses in the Bible; how many days will it require to read it through, if 30 verses are read daily? *Ans.* $1039\frac{1}{10}$ days.

9. After expending $\frac{1}{4}$ of my money, and $\frac{1}{4}$ of the remainder, I had remaining $72; how much had I at first? *Ans.* $128.

10. If I sell cloth at $1·50 per yard, and gain 25 per cent., how ought I to have sold it so as to lose 20 per cent.? *Ans.* $0·96.

11. Sold cloth at $1·50 per yard, and gained 25 per cent. What should I have lost per cent., if I had sold it at $0·96 per yard? *Ans.* 20 per cent.

12. If I buy cloth at $1·20 per yard, how must I sell it so as to gain 25 per cent.? *Ans.* $1·50.

13. A merchant has to make the following payments at three different periods: $2832 in 3 months, $2560 in 9 months, and $1450 in 16 months. The creditor wishes to receive the whole sum of $6842 at once. When ought the payment to be made? *Ans.* In 8 months.

14. A father gives to his five sons $1000, which they are to divide according to their ages, so that each elder son shall receive $20 more than his next younger brother. What is the share of the youngest? *Ans.* $160.

15. A company of 90 persons consists of men, women, and children. The men are 4 in number more than the women, the children 10 more than the adults. How many men, women, and children, are there in the company?

$$\textit{Ans.}\left\{\begin{array}{l}\text{22 men,}\\ \text{18 women,}\\ \text{50 children.}\end{array}\right.$$

16. The common school fund for the state of New-York was $1975093·15 in 1843, and during the same year there were in the state, 677995 children between the ages of 5 and 16 years. How much would the above fund amount to per scholar? Ans. $2·91, nearly.

17. The whole number of volumes in the common school libraries of New York, in 1843, was 874865. What would be their value at 37½ cents per volume?

Ans. $328074·37½.

18. The whole number of children taught in N. Y. during the year 1843, was 657782, and the whole number of schools was 10860. How many scholars on an average would each school consist of? *Ans.* Between 60 and 61.

19. Suppose the Erie canal to be 60 feet wide, and 6 feet deep; how many miles in length will it require to make one cubic mile of water? *Ans.* 77440 miles.

20. A person owning ⅗ of a copper mine, sells ¾ of his interest in it for $1800. What, at this rate, is the value of the whole? *Ans.* $4000.

21. Suppose I buy a certain lot of oranges at 3 cents apiece, and as many more at 5 cents apiece, and sell them at 4 cents apiece; do I gain or lose by the operation?

Ans. I neither gain nor lose.

22. Suppose I buy a certain number of oranges at 3 for one cent, and as many more at 5 for one cent, and sell them at 4 for one cent; do I gain or lose by the operation?

Ans. { I lose $\frac{1}{60}$ of a cent on each orange. If the whole number of oranges was 60, I should lose one cent.

23. Suppose I expend a certain sum of money for oranges at ⅓ of a cent apiece and another equal sum for another lot of oranges at ⅕ of a cent apiece, and sell them at ¼ of a ct. apiece, do I gain or lose by the operation?

Ans. I neither gain nor lose.

24. Suppose I expend a certain sum of money for oranges at 3 cents apiece, and another equal sum for another

lot at 5 cents apiece; how much do I gain on each cent expended, if I sell them at 4 cents apiece?

Ans. I gain $\frac{1}{15}$ of a cent on each cent employed in the purchase. If the whole sum employed was 15 cents, I should gain 1 cent.

25. If A can do a piece of work in 3 days, B in 4 days, and C in 5 days, how many times longer will it take B to do it alone, than it will take A and C together to do it?

Ans. $2\frac{2}{15}$ times.

26. If A can accomplish a piece of work in $\frac{1}{3}$ of a day, B in $\frac{1}{4}$ of a day, and C in $\frac{1}{5}$ of a day, how many times longer will it take B to do it alone, than it will take A and C together to do it? *Ans.* 2 times.

27. What is the shortest piece of cloth which shall be at the same time, an even number of yards, an even number of Ells Flemish, an even number of Ells English, and an even number of Ells French?

Ans. 60 quarters = 15 yards.

28. A man died, leaving $1000, to be divided between his two sons, one 14, and the other 18 years of age, in such a proportion, that the share of each being put to interest at 6 per cent., should amount to the same sum when they should arrive at the age of 21. What did each one receive?

Since the shares of each would amount to equal sums when they should come of age, it is obvious that they must have been to each other reciprocally as the amount of $1 for the respective times 7 years and 3 years. The amount of $1 for 7 years at 6 per cent., is $1·42. The amount of $1 for 3 years at 6 per cent., is $1·18. Hence, their portions were as 118 is to 142, or as 59 to 71. The sum of these numbers is 130. Therefore,

The younger must have $\frac{59}{130}$ of $1000=$453·846, nearly.
The elder must have $\frac{71}{130}$ of $1000=$546·154, nearly.

29. Divide $100 between A, B, and C, so that B may have $3 more than A, and C $4 more than B. How much must each one have?

Ans. { A has $30.
B " $33.
C " $37.

30. A can do a piece of work in 4 days, and B can do the same in 8 days. How long would it take both together to do it? *Ans.* 1⅓ days.

31. A person wishes to dispose of his horse by lottery. If he sells the tickets at $2 each, he will lose $30 on his horse; but if he sells them at $3 each, he will receive $30 more than his horse cost him. What is the value of the horse, and the number of tickets?

Ans. { Value of horse, $150.
No. of tickets, 60.

32. Thomas sold 150 pine-apples at 33⅓ cents apiece, and received the same amount of money that Henry did for water-melons at 25 cents apiece. How much money did each receive, and how many melons did Henry sell?

Ans. Each received $50, and Henry sold 200 melons.

33. A man bought apples at 5 cents a dozen, half of which he exchanged for pears, at the rate of 8 apples for 5 pears; he then sold all his apples and pears at a cent apiece, and thus gained 19 cents. How many apples did he buy, and how much did they cost?

Ans. 48 apples for 20 cents.

34. A person expended $23·40 for eggs. With one half of his money he purchased a lot at 13 cents per dozen; with the other half of his money he purchased another

lot at 9 cents per dozen. He afterward sold them all together at 11 cents per dozen. Did he gain or lose by the operation? *Ans.* He gained 80 cents.

35. Divide $1200 between A and B so that A's share may be to B's as 2 to 7. *Ans.* { A has $266⅔. B has $933⅓.

36. A gentleman spends ⅔ of his yearly income for board and lodging, ⅔ of the remainder for clothes, and ⅔ of the residue he bestows for charitable purposes, and saves $100 yearly. What is his income?

Ans. $2700.

37. If I buy an article for $4, and sell it for $5, how much per cent. do I gain? *Ans.* 25 per cent.

38. If I give $5 for an article, and sell it for $4, how much per cent. do I lose? *Ans.* 20 per cent.

39. What is the interest of $175 for 3 months, at 6 per cent.? *Ans.* $2·625.

40. How many yards of Brussels carpeting, which is ¾ of a yard wide, will it require to cover a floor 18 feet by 20 feet? *Ans.* 53⅓ yards.

41. Admitting the velocity of a cannon ball to be 1600 feet per second, what time, at this velocity, would it require to move 95 millions of miles, which is the distance from the earth to the sun, counting 365¼ days to the year.

Ans. $9\frac{12284}{13149}$ years.

42. The Winchester bushel measure is of a cylindric form, 8 inches deep, and 18½ inches in diameter, containing 2150⅖ cubic inches. What must be the side of a cubical box which shall contain the same quantity?

The cube root of 2150⅖ = 12·907, nearly, for the length *of a side*, in inches.

43. The clocks of Italy go on to 24 hours; then how

many strokes do they strike in one revolution of the index? *Ans.* 300.

44. There is an island 20 miles in circumference, and three men, A, B, and C, start from the same point, and travel the same way about it; A goes 3 miles per hour, B goes 7 miles per hour, and C goes 11 miles per hour. In what time will they all be together?

Since B gains on A 4 miles each hour, he will overtake him when he has gained the entire circumference; that is, A and B will be together at the end of every 5 hours. Again, since C gains on B 4 miles each hour, he will overtake him when he has gained the whole circumference; that is, B and C will be together at the end of every 5 hours. Consequently, they will all be together at the end of every 5 hours.

45. What is the discount of $175 for 3 months, at 6 per cent.? *Ans.* $2·586.

46. If a ship and its cargo is worth $30000, and the cargo is worth 5 times as much as the ship, what is the value of the cargo? *Ans.* $25000.

47. What is the difference between six and one half times 7, and seven and one half times 6? *Ans.* $\frac{1}{2}$.

48. Three persons, A, B, and C, form a partnership; A furnishes $1000, B $600, and C $450; at the end of 6 months, C withdraws his capital, but no dividend is made until the end of the year, when it is found that the firm has gained $244·16. How is this gain to be divided between the partners?

Ans. A has $\frac{40}{73}$ of $244·16 = $133·79 −
B has $\frac{24}{73}$ of $244·16 = $ 80·27 +
C has $\frac{9}{73}$ of $244.16 = $ 30·10 +

Proof $244·16.

49. Three persons, A, B, and C, engage to build a certain piece of wall for $244·16. While A can build 10 rods, B can build but 6, and C but $4\frac{1}{2}$. When the wall is half completed, C ceases to labor upon it, and A and B finish it. What part of the $244·16 ought each to receive?

Ans. A ought to have $135·85.
B " " " $ 81·51.
C " " " $ 26·80.

50. A and B together can build a wall in 4 days, A and C can together build it in 5 days, B and C can together build it in 6 days. What time would it require for all together to accomplish it?

A and B can in one day build $\frac{1}{4}$ of it $=\frac{15}{60}$ of it.
A and C " " " " " $\frac{1}{5}$ of it $=\frac{12}{60}$ of it.
B and C " " " " " $\frac{1}{6}$ of it $=\frac{10}{60}$ of it.

The sum of these fractions, $\frac{15}{60}+\frac{12}{60}+\frac{10}{60}=\frac{37}{60}$, is evidently twice the fractional part accomplished by all in one day. Hence, they all would in one day accomplish $\frac{1}{2}$ of $\frac{37}{60}=\frac{37}{120}$; consequently, in $\frac{120}{37}=3\frac{9}{37}$ days, they would finish it.

50. A note of $10000 given Jan. 1st, 1840, has received the following indorsements: January 1st, 1841, indorsed $2952·28, January 1st, 1842, indorsed $2952·28, January 1st, 1843, indorsed $2952·28. How much remained due January 1st, 1844, interest being computed at 7 per cent.?

Ans. There was due $2952·28.

51. Two hunters, A and B, kill a deer, whose weight they are desirous of knowing. For this purpose, they rest a stick across the limb of a tree; then suspending the deer at the shorter extremity, they find that its weight is just counterpoised by the weight of A, who suspends himself by his hands at the other extremity. Without changing

the point of support of the stick, they take the deer from the shorter extremity and suspend it at the longer extremity of the stick, when it was found to be exactly balanced by B's weight, when suspended at the shorter extremity of the stick. Now, supposing A to weigh 147 pounds, and B to weigh 192 pounds, what must have been the weight of the deer?

By the principle of the lever, we know that when different weights at its extremities balance each other, they are to each other inversely as the lengths of the arms to which they are attached. Hence, in the first experiment, we know that the weight of A is to the deer's weight, as the shorter arm is to the longer arm. In the second experiment, the deer's weight is to B's weight, as the shorter arm is to the longer arm. Consequently, A's weight is to the deer's weight, as the deer's weight is to B's weight; that is, the deer's weight is a mean proportional between A's weight and B's weight. Therefore, if we multiply the number of pounds which A weighed, by the number of pounds which B weighed, and extract the square root of the product, it will give the weight of the deer in pounds.

$$147 \times 192 = 28224.$$

And $\sqrt{28224} = 168$ the weight of the deer in pounds.

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